CHAPTER-05

THE VIBRATIONS OF ELECTRO-MAGNETO NONLOCAL THERMOELASTIC SPHERE WITH VOIDS AND THREE-PHASE-LAG MODEL

5.1 Introduction

In this chapter the transversely isotropic electro-magneto nonlocal thermoelastic hollow sphere with voids material has been addressed for free vibration analysis. By using time harmonics, the stress-strain relations and modeling equations have been transformed into ordinary differential equations. The unknown field functions have been eliminated by using matrix elimination technique. In order to investigate the vibration analysis, the relations of frequency equations have been solved for assumed boundary conditions. To authenticate the phase-lag effects on the model of generalized thermoelasticity, the analytical results have been shown graphically in absence/presence of magnetic field. The magneto-thermoelastic solid materials with voids in respect of analysis of free vibrations have many applications such as designers of new materials in practical situations, acoustics, and oil prospecting etc.

5.2 The Basic Fundamental Equations and Mathematical Model

In this chapter we consider a thermally conducted nonlocal magneto-thermoelastic transversely isotropic elastic hollow sphere/disk with voids material of three-phase-lag (TPL) model in the reference of generalized thermoelasticity with the domain $a \le r \le a\eta$. The field components are assumed as temperature component $T = T(r,t)$, concentration of voids volume fraction $\varphi = \varphi(r,t)$ and displacement vector $\boldsymbol{u} = (u_r, u_\theta, u_\phi) = (u(r,t), 0, 0)$. The strain components of sphere are $e_{rr} = \dfrac{\partial u}{\partial r}, e_{\theta\theta} = e_{\theta\theta} = \dfrac{u}{r}$, $e_{r\theta} = 0 = e_{r\phi} = e_{\theta\phi}$. The Maxwell's equations and the generalized Ohm's law have been generated by electro-magnetic field in the absence of charge density and displacement current as:

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}, \quad \nabla \times \boldsymbol{H} = \boldsymbol{J}, \quad \boldsymbol{B} = \mu_e \boldsymbol{H}, \quad \nabla.\boldsymbol{B} = 0, \; \boldsymbol{J} = \sigma(\boldsymbol{E} + \dot{u} \times \boldsymbol{B}).$$

(5.1)

Here the strength of magnetic field is $\boldsymbol{H} = \boldsymbol{H}_0 + h$, where $\boldsymbol{H}_0 = (0, 0, H_0)$, $\boldsymbol{J}$ is current density which is neglected due to small effect of temperature gradient, h is perturbation of magnetic field which is considered so small that the product of displacement vector (u) and perturbation of magnetic field (h) and their derivatives neglected due to linearization of basic equations. Therefore, following Dhaliwal and Singh (1980) and Cowin and Nunziato (1983), the nonlocal stress-strain temperature relations and governing fundamental equations without heat sources, body forces and free from voids concentration are given as :

$$\sigma_{ij,j} + F_i = \rho(1-\zeta^2\nabla^2)\frac{\partial^2 u}{\partial t^2}, \tag{5.2}$$

$$-be + \alpha\nabla^2\varphi - \left(\xi_1 + \xi_2\frac{\partial}{\partial t}\right)\varphi + \rho\,\chi(1-\zeta^2\nabla^2)\frac{\partial^2\varphi}{\partial t^2} + MT = 0, \tag{5.3}$$

$$\begin{aligned}&\left(\frac{\partial^2}{\partial t^2} + t_q\frac{\partial^3}{\partial t^3} + \frac{t_q^2}{2}\frac{\partial^4}{\partial t^4}\right)\left(\rho C_e T + T_0\left(\beta_r\frac{\partial u}{\partial r} + \beta_\theta\frac{u}{r}\right) + MT_0\varphi\right)\\ &= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)\left(K\left(\frac{\partial}{\partial t} + t_T\frac{\partial^2}{\partial t^2}\right) + K^*\left(1 + t_v\frac{\partial}{\partial t}\right)\right)\end{aligned}, \tag{5.4}$$

$$(1-\zeta^2\nabla^2)\sigma_{ij} = \sigma_{ij}^L = c_{ij}e_{kl} + b_{ij}\varphi - \beta_{ij}T \quad ;(i,j=r,\theta), \tag{5.5}$$

Here $\beta_{ij}\,;(i,j=r,\theta)$ is the components of thermal modulii where $\beta_r = \beta_\theta = (c_{11}+c_{12})\alpha_1 + c_{13}\alpha_3$, σ_{ij} and $e_{ij}\,;(i,j=r,\theta)$ are stresses and strains; $\alpha_1 = \alpha_3 = \alpha_T$ is coefficient of linear thermal expansion (Dhaliwal and Singh (1980)), T is assumed as increase in the reference temperature T_0 of the medium, $b_{ij} = b$ are the voids parameters, φ is the void volume fraction, χ is the equilibrated inertia, ξ_1, ξ_2 are the material constants because of presence of voids, t_T, t_q, t_V represent the Phase-lags temperature gradient, heat flux, thermal displacement gradient respectively. $\boldsymbol{F}_i$ is the body force $\boldsymbol{F} = (\boldsymbol{J}\times\boldsymbol{B})$. If ν is Poisson ratio and E is Young's modulus, then elastic constants are

$$c_{11} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}, \quad c_{12} = \frac{E\nu}{(1+\nu)(1-2\nu)},$$

and Laplacian operator is $\nabla^2 = \dfrac{\partial^2}{\partial r^2} + \dfrac{2}{r}\dfrac{\partial}{\partial r}$.

Substituting the values of stresses from equation (5.5) in equation (5.2) we obtained as

$$\frac{1}{c_{11}}\boldsymbol{F}_{r,r}+\left(\frac{\partial^2 u}{\partial r^2}+\frac{2}{r}\frac{\partial u}{\partial r}-\frac{2u}{r^2}\right)-\frac{\rho}{c_{11}}(1-\zeta^2\nabla^2)\frac{\partial^2 u}{\partial t^2}+\frac{b}{c_{11}}\frac{\partial\varphi}{\partial r}-\frac{\beta_r}{c_{11}}\frac{\partial T}{\partial r}=0. \quad (5.6)$$

The analysis has been considered to be restricted to thermoelastic sphere in radial direction, then using equations (5.1) in Lorentz force i.e. $F_r=(\boldsymbol{J}\times\boldsymbol{B})_{,r}$ in radial direction, we obtained

$$F_\phi=0\,,\quad F_\theta=0,\;\; F_r=\mu_e H_0^2\left(\frac{2u}{r}+\frac{\partial u}{\partial r}\right). \quad (5.7)$$

Substituting Lorentz force value from equation (5.7) in equation (5.6), we get

$$\frac{\mu_e H_0^2}{c_{11}}\frac{\partial}{\partial r}\left(\frac{\partial u}{\partial r}+\frac{2u}{r}\right)+\left(\frac{\partial^2 u}{\partial r^2}+\frac{2}{r}\frac{\partial u}{\partial r}-\frac{2u}{r^2}\right)-\frac{\rho(1-\zeta^2\nabla^2)}{c_{11}}\frac{\partial^2 u}{\partial t^2}+\frac{b}{c_{11}}\frac{\partial\varphi}{\partial r}-\frac{\beta_r}{c_{11}}\frac{\partial T}{\partial r}=0, \quad (5.8)$$

Taking divergence to both sides of equation (5.8), we obtained

$$R_h\nabla^2 e-\frac{\rho(1-\zeta^2\nabla^2)}{c_{11}}\frac{\partial^2 e}{\partial t^2}+\frac{b}{c_{11}}\nabla^2\varphi-\frac{\beta_r}{c_{11}}\nabla^2 T=0\,, \quad (5.9)$$

where $e=\dfrac{1}{r^2}\dfrac{\partial}{\partial r}(r^2 u),\;\; R_h=1+\dfrac{\mu_e H_0^2}{c_{11}}.$

The traction free isothermal and thermally insulated boundary conditions of TPL model of generalized nonlocal magneto-thermoelastic hollow sphere with voids have been considered with domain $a\le r\le a\eta$ given below:

$$\begin{cases}\dfrac{\partial T}{\partial r}=0\;,\quad \sigma_{rr}=0\,,\;\varphi=0\quad;\quad r=a\;,\;\; r=a\eta\\ T=0\quad,\quad \sigma_{rr}=0\,,\;\varphi=0\quad;\quad r=a\;,\;\; r=a\eta\end{cases}, \quad (5.10)$$

5.3 Solution of the Mathematical Model

We set up the following non-dimensional parameters

$$\left.\begin{array}{l}(u',r',\zeta_0)=\dfrac{1}{a}(u,r,\zeta)\,,(\tau,\tau_T,\tau_q,\tau_v)=\dfrac{c}{a}(t,t_T,t_q,t_v),\,(\tau_{RR},\tau_{\theta\theta})=\dfrac{1}{c_{11}}(\sigma_{rr},\sigma_{\theta\theta})\,,\theta=\dfrac{T}{T_0},\\ c_0=\dfrac{c_{12}}{c_{11}},\,c=\sqrt{\dfrac{c_{11}}{\rho}},\;\;\phi=\dfrac{\chi\Omega^{*2}}{a^2}\varphi,\;\;\bar{\xi}=\dfrac{c}{a}\dfrac{\xi_2}{\xi_1},\;\;\bar{\beta}_R=\dfrac{\beta_r T_0}{c_{11}},\;\;\bar{\beta}_\theta=\dfrac{\beta_\theta T_0}{c_{11}},\;\;\bar{b}^*=\dfrac{a^2\bar{b}}{\chi\Omega^{*2}},\,\bar{b}=\dfrac{b}{c_{11}}\end{array}\right\}. \quad (5.11)$$

The dashes have been suppressed for convenience. Using non-dimensional constants as proposed in equation (5.11) in equations (5.3) - (5.5) and (5.9), we attain equations in non-dimensional form:

$$R_h\nabla_R^2 e+\bar{b}^*\nabla_R^2\phi-\bar{\beta}_R\nabla_R^2\theta=(1-\zeta_0^2\nabla_R^2)\frac{\partial^2 e}{\partial\tau^2}, \tag{5.12}$$

$$-a_2 e+\nabla_R^2\phi-a_1\left(1+\bar{\xi}\frac{\partial}{\partial\tau}\right)\phi+a_3\theta=(1-\zeta_0^2\nabla_R^2)\frac{1}{\delta_1^2}\frac{\partial^2\phi}{\partial\tau^2}, \tag{5.13}$$

$$\left(\frac{\partial^2}{\partial\tau^2}+\tau_q\frac{\partial^3}{\partial\tau^3}+\frac{\tau_q^2}{2}\frac{\partial^4}{\partial\tau^4}\right)\left(\Omega^*\theta+a_4 e+a_5\phi\right)=\left(\left(\frac{\partial}{\partial\tau}+\tau_T\frac{\partial^2}{\partial\tau^2}\right)+\bar{K}\left(1+\tau_v\frac{\partial}{\partial\tau}\right)\right)\left\{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\theta}{\partial r}\right)\right\}, \tag{5.14}$$

$$\left.\begin{aligned}(1-\zeta_0^2\nabla_R^2)\tau_{RR}&=\tau_{RR}^L=e+2(c_0-1)\frac{U}{r}+\bar{b}^*\phi-\bar{\beta}_R\theta\\(1-\zeta_0^2\nabla_R^2)\tau_{\theta\theta}&=\tau_{\theta\theta}^L=c_0 e+(1-c_0)\frac{U}{r}+\bar{b}^*\phi-\bar{\beta}_\theta\theta\end{aligned}\right\}, \tag{5.15}$$

where $a_1=\dfrac{\xi_1 a^2}{\alpha}, a_2=\dfrac{b\chi\Omega^{*2}}{\alpha}, a_3=\dfrac{M\chi\Omega^{*2}T_0}{\alpha}, a_4=\dfrac{\varepsilon_T\Omega^*}{\bar{\beta}_R}, a_5=\dfrac{Mca^3}{K_\theta\chi\Omega^{*2}}, \omega^*=\dfrac{c_{11}C_e}{K_\theta}$,

$\bar{K}=\dfrac{aK^*}{cK}, \Omega^*=\dfrac{a\omega^*}{c}, \varepsilon_T=\dfrac{T_0\beta_r^2}{\rho C_e c_{11}}, \delta_1^2=\dfrac{\alpha}{\chi c_{11}}, e=\dfrac{1}{r^2}\left(\dfrac{\partial}{\partial r}(r^2U)\right), \nabla_R^2=\dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2\dfrac{\partial}{\partial r}\right)$

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Now we propose the time harmonics as

$$\begin{pmatrix}\bar{e}\\\bar{\phi}\\\bar{\theta}\end{pmatrix}=\begin{pmatrix}e\\\phi\\\theta\end{pmatrix}exp(i\Omega\tau). \tag{5.16}$$

Here $\Omega=\dfrac{\omega a}{c}$ is the circular frequency. Using equation (5.16) in equations (5.12–5.14), we get

$$\begin{pmatrix}(\nabla_R^2+A_{11}) & A_{12}\nabla_R^2 & -A_{13}\nabla_R^2\\-A_{21} & (\nabla_R^2+A_{22}) & A_{23}\\A_{31} & A_{32} & (\nabla_R^2-A_{33})\end{pmatrix}\begin{pmatrix}\bar{e}\\\bar{\phi}\\\bar{\theta}\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}, \tag{5.17}$$

where

$$A_{11}=\frac{\Omega^2}{R_h-\zeta_0^2\,\Omega^2},\; A_{12}=\frac{\bar{b}^*}{R_h-\zeta_0^2\,\Omega^2},\; A_{13}=\frac{\bar{\beta}_R}{R_h-\zeta_0^2\,\Omega^2},\; A_{21}=\frac{a_2}{a_1^*},\; A_{22}=\frac{a_2^*}{a_1^*},\; A_{23}=\frac{a_3}{a_1^*},$$

$$a_1^*=\frac{\delta_1^2-\zeta_0^2\,\Omega^2}{\delta_1^2},\; a_2^*=\frac{a_1 i\,\Omega\,\bar{\xi}^*\,\delta_1^2+\Omega^2}{\delta_1^2},\; A_{31}=\frac{a_4^*}{a_7^*},\; A_{32}=\frac{a_5^*}{a_7^*},\; A_{33}=\frac{a_6^*}{a_7^*},$$

$$a_4^*=\Omega^2\,\Omega^*\,\tau_q^*,\;\; a_5^*=\Omega^2\,\tau_q^*\,a_4,\;\; a_6^*=\Omega^2\,\tau_q^*\,a_5$$

$$a_7^*=(\Omega^2\tau_T^*-\bar{K}\,i\,\Omega\,\tau_v^*),\;\; \bar{\xi}^*=i\,\Omega^{-1}-\bar{\xi},\;\; \tau_q^*=\left(\Omega^{-2}+i\Omega^{-1}\tau_q-\frac{\tau_q^2}{2}\right),$$

$$\tau_T^*=i\,\Omega^{-1}-\tau_T\;,\;\tau_v^*=i\,\Omega^{-1}-\tau_v$$

For the solution of equations (5.17), we have non-trivial solution as given:

$$(\nabla_R^6-L^*\nabla_R^4+M^*\nabla_R^2-N^*)(\bar{e}\,,\,\bar{\phi},\,\bar{\theta})=0\,, \tag{5.18}$$

where $L^*=\left(A_{33}-A_{11}-A_{22}-A_{13}A_{31}+A_{12}A_{21}\right)$,

$$M^*=\left(A_{12}A_{21}A_{33}+A_{13}A_{21}A_{32}+A_{13}A_{31}A_{22}+A_{12}A_{23}A_{31}-A_{11}A_{33}+A_{11}A_{22}-A_{23}A_{32}-A_{22}A_{33}\right),$$

$$N^*=A_{11}A_{23}A_{33}+A_{11}A_{22}A_{33}\,.$$

Because the solution of equation (5.18) is noted to be bounded for $r\to\infty$, therefore for the bounded conditions, the roots must be real and positive, i.e. $\mathrm{Re}(k_1,k_2\,,\,k_3)\ge 0$. Therefore, the roots $k_i\,;\; i=1,\,2,\,3$ of equation (5.18) are:

$$k_1=\sqrt{\frac{1}{3}\left(2p_1\sin p_2+L^*\right)},\; k_2=\sqrt{\frac{1}{3}\left(L^*-p_1(\sqrt{3}\cos p_2+\sin p_2)\right)},\; k_3=\sqrt{\frac{1}{3}\left(L^*+p_1(\sqrt{3}\cos p_2-\sin p_2)\right)},$$

where, $p_1=\sqrt{L^{*2}-3M^*}$, $p_3=-\dfrac{2L^{*3}-9L^*M^*+27N^*}{2L^{*3}}$, $p_2=\dfrac{1}{3}\sin^{-1}(p_3)$.

Since on splitting the equation (5.18), we find the equations in Bessel form and the solution might be written as

$$\begin{pmatrix}\bar{\theta}\\ \bar{e}\\ \bar{\phi}\end{pmatrix}=\frac{1}{\sqrt{r}}\sum_{i=1}^{3}\begin{pmatrix}1\\ R_i\\ S_i\end{pmatrix}\left(P_iJ_{1/2}(k_ir)+Q_iY_{1/2}(k_ir)\right), \tag{5.19}$$

where, $R_i = \dfrac{k_i^2(A_{13}A_{21} - A_{23}) - A_{11}A_{23}}{A_{13}k_i^4 + k_i^2(A_{13}A_{22} + A_{12}A_{23})}$, $S_i = -\dfrac{A_{31}k_i^2 + A_{22}A_{31} + A_{21}A_{32}}{k_i^4 + (A_{22} - A_{33})k_i^2 - (A_{22}A_{33} + A_{23}A_{32})}$.

Here P_i, Q_i; $i = 1, 2, 3$ are constants that depend on Ω only. Here $J_{1/2}$ and $Y_{1/2}$ are Bessel functions of First and Second kinds of order half. Resolving cubical dilation $(\bar{e})$ from second part of equation (5.19) for displacement $\bar{u}$, we obtain

$$\bar{u} = \frac{1}{\sqrt{r}} \sum_{i=1}^{3} \frac{1}{k_i} R_i \left(P_i J_{3/2}(k_i r) - Q_i Y_{3/2}(k_i r) \right). \tag{5.20}$$

5.4 Frequency Relations

For the analysis of stress free vibrations of three-phase-lag model, the frequency equations are obtained by substitution of equations (5.19–5.20) in the boundary conditions given in equations (5.10), at inner and outer radii $r = 1$ and $r = \eta$. On simplification of these equations, we obtain a system of homogenous equations given below:

$$(\Upsilon_{ij})_{6\times 6} (\Pi)_{6\times 1} = 0 \; ; \; i, j = 1 \; to \; 6, \tag{5.21}$$

where $\Pi = (P_1, P_2, P_3, Q_1, Q_2, Q_3)^T$. From Equation (5.21) we obtain six homogeneous linear equations with six unknowns. Hence, for the non-trivial solution of equation (5.21), we must have

$$\left| \Upsilon_{ij} \right| = 0 \; ; \quad i, j = 1 to \; 6, \tag{5.22}$$

From equation (5.22) the constant parameters Υ_{ij} ; $i, j = 1 to \; 6$ have been defined in thermally insulated boundary conditions in set I and isothermal boundary conditions in set II given below:

Set I: The parameters of Υ_{ij} ; $i, j = 1 to \; 6$ are

$$\left.\begin{aligned}
&\Upsilon_{1j} = \mathrm{H}_i J_{1/2}(k_i) + 2\left((c_0 - 1)/k_i\right) R_i J_{3/2}(k_i); \; i, j = 1 \text{ to } 3; \\
&\Upsilon_{1j} = \mathrm{H}_i Y_{1/2}(k_i) - 2\left((c_0 - 1)/k_i\right) R_i Y_{3/2}(k_i); \; i = 1, 2, 3, \; j = 4, 5, 6 \\
&\Upsilon_{3j} = S_i J_{1/2}\left(k_i\right); i, j = 1, 2, 3; \; \Upsilon_{3j} = S_i Y_{1/2}\left(k_i\right); \; i = 1, 2, 3; \; j = 4, 5, 6; \\
&\Upsilon_{5j} = k_i J_{3/2}\left(k_i\right); i, j = 1, 2, 3; \Upsilon_{5j} = -k_i Y_{3/2}\left(k_i\right); i = 1, 2, 3; \; j = 4, 5, 6;
\end{aligned}\right\}. \tag{5.23}$$

Set II: In this case, the elements of $\Upsilon_{1j}, \Upsilon_{2j}, \Upsilon_{3j}, \Upsilon_{4j}; j = 1 \text{ to } 6$, remain same as given in equation (5.23). The remaining elements of $\Upsilon_{5j}, \Upsilon_{6j}; j = 1 \text{ to } 6$ in equation (5.22) are given below

$$\left.\Upsilon_{5j} = J_{1/2}(k_i);\ i, j = 1,2,3,\quad \Upsilon_{5j} = Y_{1/2}(k_i);\ i = 1,2,3;\ j = 4,5,6;\right\}. \qquad (5.24)$$

The constant elements of $\Upsilon_{2j}, \Upsilon_{4j}, \Upsilon_{6j};\ j = 1 \text{ to } 6$ are obtained by inserting η along with k_i, in elements of $\Upsilon_{1j}, \Upsilon_{3j}, \Upsilon_{5j};\ j = 1 \text{ to } 6$.

5.5 Deduction of Analytical Results

If we assume $\zeta_0 = 0$, then the present analysis is reduced to transversely isotropic TPL electro magneto thermoelastic voids hollow sphere. Again, if we thermal equilibrium has been established and the following constants are ignored, i.e. $\zeta_0 = 0$, $\alpha = b = M = \xi_1 = \xi_2 = 0 \quad t_q = t_v = t_T = 0$, the present analysis is reduced to classical magneto-thermoelastic sphere. If the thermal and thermo-mechanical constants are removed $\beta_R = \varepsilon_T = 0 = T$, then the analysis has been reduced to transversely isotropic elastic sphere.

5.6 Numerical Results and Discussion

For the validation of analytical results, computations have been proposed for TPL model of magneto-thermoelastic hollow sphere with voids in nonlocal elasticity. The simulated results are performed for generalized thermoelastic models, i.e. Lord-Shulman (LS), dual-phase-lag (DPL), three-phase-lag (TPL) and coupled thermoelasticity (CTE) in absence/presence of magnetic fields for nonlocal voids thermoelastic hollow sphere by taking the normalized thickness of the disk $\eta = 2.0$. For computation purpose the transversely isotropic material of single crystal of zinc thermoelastic solid with voids material has been assumed and its constant values are given in SI units (Chadwick and Seet (1970))

$$C_e = 3.9 \times 10^2\, JKg^{-1} \deg^{-1}, \rho = 7.14 \times 10^3\, Kg\, m^{-3}, T_0 = 296K,\ \omega = 10, \chi = 1.753 \times 10^{-15} m^2,$$
$$\alpha = 3.688 \times 10^{-5} N, c_{11} = 1.628 \times 10^{11}\, Nm^{-2},\ c_{12} = 1.562 \times 10^{11}\, Nm^{-2},$$
$$K = 1.24 \times 10^2 Wm^{-1} \deg^{-1}, \beta_r = \beta_\theta = 5.75 \times 10^6\, Nm^{-2} \deg^{-1}, M = 2.0 \times 10^6\, Nm^{-2} \deg^{-2},$$
$$\xi_1 = \xi_2 = 1.475 \times 10^{10}\, Nm^{-2},\ b = 1.13849 \times 10^{10}\, Nm^{-2}.$$

The magnetic field parameters have been assumed as $\mu_e = 4\pi \times 10^7 H/m,\ H_0 = 10^8 A/m$

from Othman and Hilal (2017). The nonlocal parameter and three-phase-lag (TPL) parameter values have been considered as $\xi_0 = 2.3102$ and $t_v = 0.05$, $t_T = 0.07$, $t_q = 0.09$, $K^* = 7.0$. The numerically analyzed computations are employed to equation (5.22) for thermally insulated cases.

The numerically analyzed values for the computations of frequency equation (5.22) of Ω might be written as $\Omega^m = \Omega_R^m + i\,\Omega_I^m$. The real and imaginary parts have been considered as natural frequencies $\Omega_R^m = \Omega_R$ and dissipation factor $\Omega_I^m = \Omega_I$ correspondingly. The value m has been considered as mode number denoted the root of equation (5.22). The numerically simulated values have been presented graphically for TPL, DPL, LS and CTE theories of magneto nonlocal thermoelastic hollow sphere in absence and presence of magnetic field. The real part i.e. natural frequencies against mode number have been represented for nonlocal thermoelastic voids sphere with and without magnetic field at $\eta = 2.0$ in Fig. 5.1(a–b). It is concluded from Fig. 5.1(a-b) that initially the variation of vibrations is low and with increasing values of m , the variation of vibrations go on increasing with increasing mode number.

Fig. 5.2(a-b) has been presented for imaginary part i.e. dissipation factor (Ω_I) versus mode number (m) for nonlocal thermoelastic voids sphere with and without magnetic field at $\eta = 2.0$. This is observed from Fig. 5.2 that initially dissipatioleft to right, the dissipated vibrations go on increasing. The frequency shift (Ω_{shift}) of transversely isotropic electro-magneto generalized thermoelastic hollow sphere has been

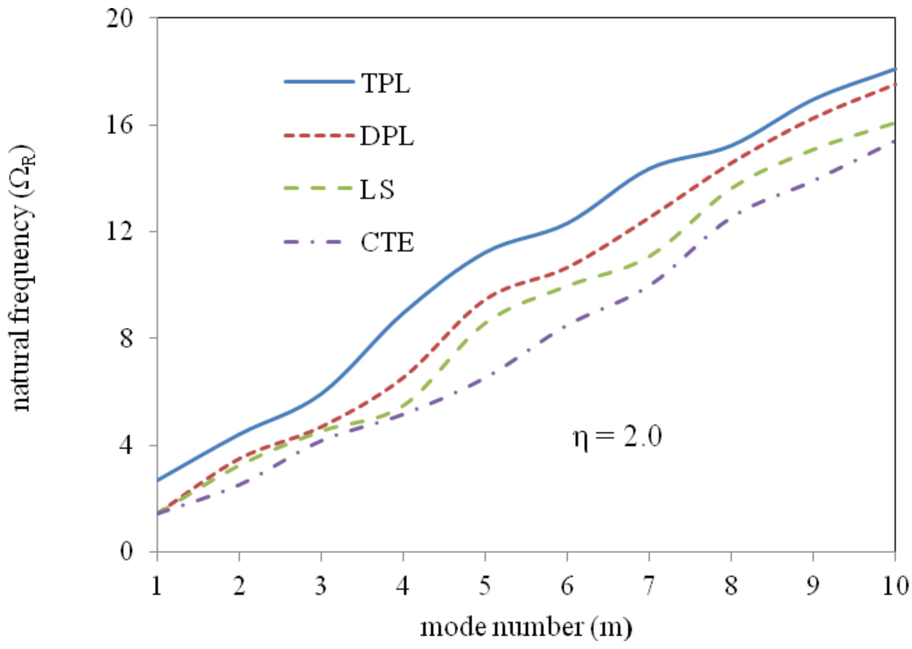

Figure 5.1(a): Natural frequency (Ω_R) against mode number (m) for TPL, DPL, LS and CTE models of nonlocal thermoelastic sphere with voids with magnetic field.

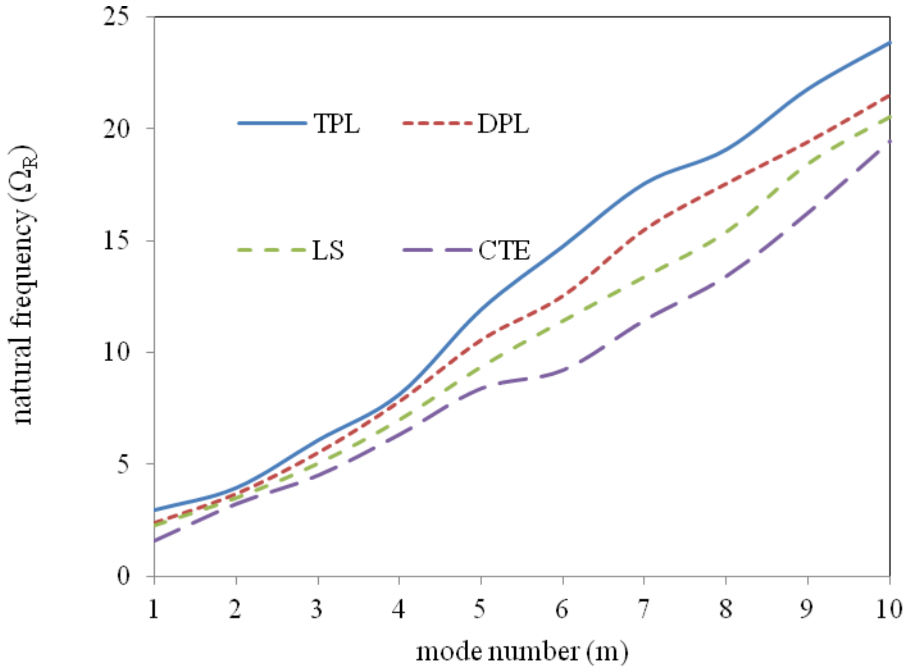

Figure 5.1(b): Natural frequency (Ω_R) against mode number (m) for TPL, DPL, LS and CTE models of nonlocal thermoelastic sphere with voids without magnetic field.

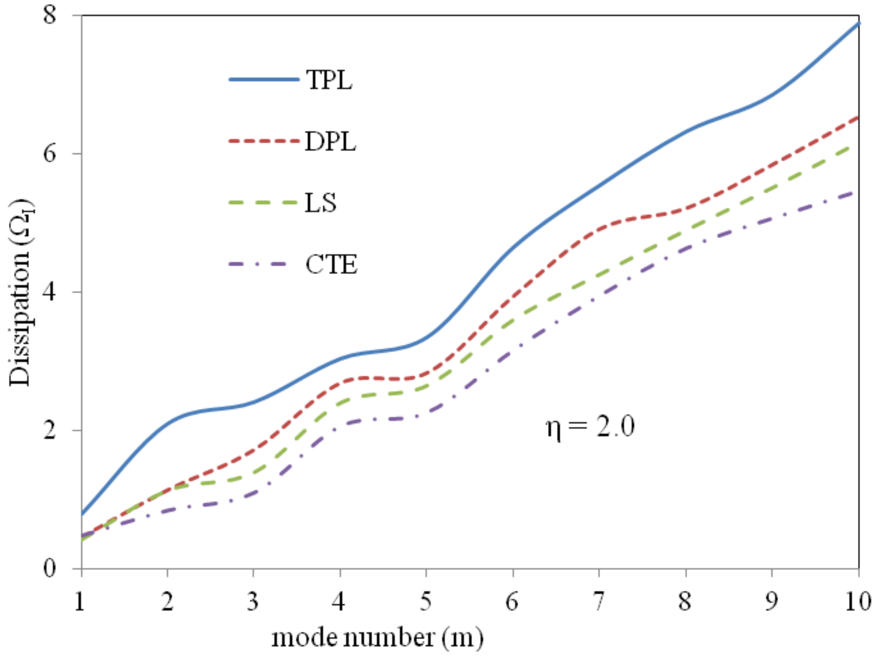

Figure 5.2(a): Dissipation (Ω_I) versus mode number (m) for TPL, DPL, LS and CTE models of nonlocal thermoelastic sphere with voids with magnetic field.

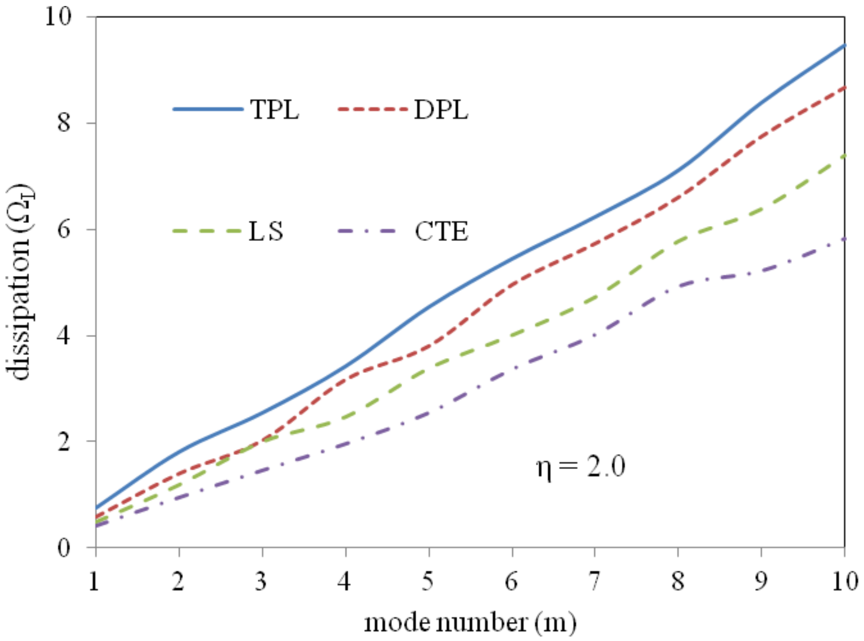

Figure 5.2(b): Dissipation (Ω_I) versus mode number (m) for TPL, DPL, LS and CTE models of nonlocal thermoelastic sphere with voids without magnetic field.

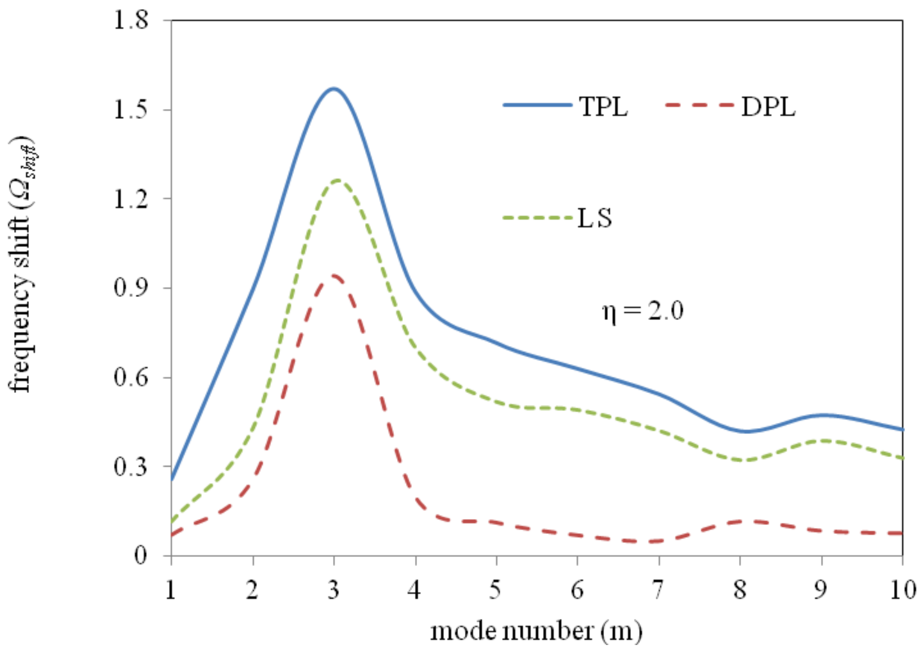

Figure 5.3(a): Frequency shift (Ω_{shift}) versus mode number (m) for TPL, DPL, LS and CTE models of nonlocal thermoelastic sphere with voids with magnetic field.

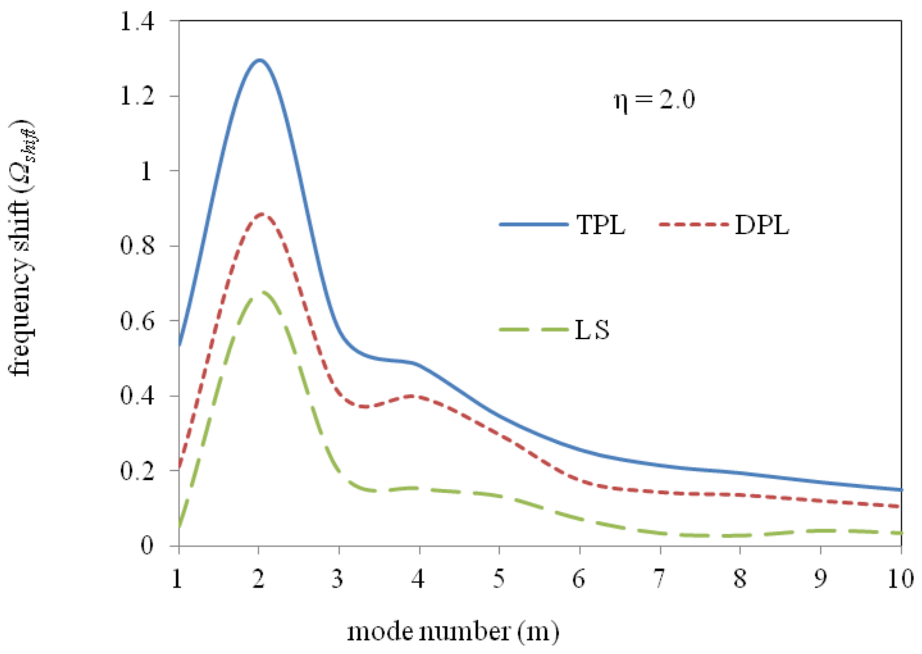

Figure 5.3(b): Frequency shift (Ω_{shift}) versus mode number (m) for TPL, DPL, LS and CTE models of nonlocal thermoelastic sphere with voids without magnetic field.

calculated from Sharma et al. (2021b) as: $\Omega_{shift} = \left|(\Omega_R^{M^*} - \Omega_R^{CTE}) / \Omega_R^{CTE}\right|$, where M^* assumes for TPL, DPL, LS models of generalized nonlocal thermoelastic sphere with voids, plotted for frequency shift (Ω_{shift}) against mode number (m) at normalized thickness $\eta = 2.0$ for in presence/absence of magnetic field. It is observed from Fig. 5.3(a–b) that the behavior of vibrations are low initially, attain peak values at $m = 3.0$ and $m = 2.0$, with increasing values of m the vibrations go on decreasing to become linear at $m = 4.0$. This is noticed from all the figures that the vibrations are larger in case of TPL model in comparison with DPL, LS and CTE models of thermoelasticity. Also due to the effect of magnetic field the behavior is noted to be larger in absence of magnetic field in contrast to presence of magnetic field.

5.7 Conclusions

The transversely isotropic nonlocal electro-magneto elastic hollow sphere with voids has been presented for TPL model of generalized thermoelasticity. The relations of frequency equations have been derived and examined computationally for analytical results. The effects of magnetic field clearly designates that the variations are larger in absence of magnetic field in contrast to presence of magnetic field. It is seen in first figure that dissipations and natural frequencies go on increasing with increasing mode number. This study may find useful applications for those researchers and scientists who are working in the field of seismology for drilling, mining in the earth's crust and porous materials.