# CHAPTER-01

# INTRODUCTION

## 1.1 Preliminary

There exist an infinite number of very minute particles which possesses space and dimensions in static and dynamic bodies. By virtue of intermolecular forces these particles occupy shape and size due to action of forces. If there is a very small distance between two neighboring particles as compared to the total dimension of the body, then that state is known as continuum body state. Hence, Continuum Mechanics is one of the branch of Mechanics which deals with mechanical as well as kinematics behavior of the continuous matter. In the present scenario of science and technology, the materials play a highly significant role in different types of mathematical applications to develop different mathematical models. One cannot find so easily these types of applications in engineering and science so far. In engineering, the materials used are single crystals and polycrystalline type solids, having controlled purity. In last decade solutions have been obtained from thermoelastic and generalized thermoelastic contact problems, where the distribution of the stresses has been improved which is caused by the frictional heat generation. In most structural components, different types of engineering materials and their vibrations have many practical applications in fields like geophysics, aerospace and navigations.

## 1.2 Elasticity

Deformation is that state of body in which there is relative change in the positions of the particles when they have been conquered to external forces and changed into strained state. The property of solid materials by which the external forces are removed, regains its original size or shape is called elasticity. All bodies present in nature may be less or more elastic or deformable. The bodies in which there is very less or small deformation such that it can be neglected is known as rigid body. The medium in which all the properties remain same in all directions is known as isotropic elastic medium. Glass is a good example of this medium. If the elastic properties of a solid medium at a given point vary along with the direction, then that medium is termed as anisotropic medium. Wood is a fine example of anisotropic medium. A solid medium, in which material possesses same properties at all points, is called homogeneous medium. Steel is the example of homogenous medium. The tensor quantity stress is defined as the ratio of the internal body forces to the external load applied, while strain is termed as the measure of deformation in the material. The stress tensor is completely specified by nine components and due to symmetry, only six independent components can completely specify the state of stress at any point of the body. Sokolnikoff (1946) gave complete details about the state of stress at a point. Robert Hooke gave Hooke's law which describes the relationship between stress and strain of the medium within elastic limit. By Robert Hooke, it is stated as "*ut tensio sic vis*" which means that 'extension is directly proportional to force'. Sir Raman (1955) explained that the relationship about stress and strain which vary from one point to another point, expressed in dynamic and static states. Generally, 81 elastic constants are involved in stress-strain proportionality, because of symmetry they reduces to 36 in case of triclinic crystal for the elastic behavior. Raman and Vishwanathan (1955) and Raman and Krishnamurthy (1955) studied the behavior of elastic constants in addition to investigate the propagation of waves and vibrations. Huang et al. (2010) analyzed two dimensional elastic problem of stress formulation which was solved by expansion method in eigen functions technique. Natural frequencies and plane transverse oscillations in elastic medium of inhomogeneous rod with varying stiffness, were studied by Akulenko and Nesterov (2012). Lata and Singh (2020) studied the effect of non-locality in two dimensional isotropic thermoelastic solids due to deformation theory. Pascale and Kumar (2022) represented elasto-plastic model for the behavior of materials which was extended in plastic slip by using degradation of shear stresses.

## 1.3 Thermoelasticity

It is the study of the relationship between the elastic properties of the material and its temperature or relationship between its thermal conductivity and stresses. The thermo-dynamical changes in equilibrium state are interlinked with external work and exchange of heat etc. The term classical dynamic elasticity was developed under the simplified assumption that heat exchange between the parts of the body due to heat conduction occurs slowly and hence motion may be regarded as adiabatic. Therefore thermoelasticity comprises with the theory of heat conduction, the theory of thermal stresses, temperature distribution produced by deformation and thermoelastic dissipation. The coupling between thermal and strain fields give rise to the coupled theory of thermoelasticity. The change in body temperature is caused not only due to the external and internal heat sources, but also by the process of deformation itself. This change in temperature is very small in classical thermoelasticity and thus inertia term in the elastic equation of motion and coupling term in heat conduction equation seems to be neglected. The theory of stress firstly originated by Duhamel (1837) in which the relationship between distribution of strains and temperature gradient in heat conduction equation was proved. Neumann (1885), Voigt (1887) and Muller (1971) proved thermodynamical justification of equations given by Duhamel (1837). Boit (1956) explained the heat conduction equation derivation by including the dilatation term in thermodynamical irreversible processes. Sarkar and Lahiri (2012) proposed the three-dimensional isotropic thermoelastic half-space problem in a homogeneous medium, subjected to traction free, time-dependent heat source. Sharma et al. (2012) studied the problem of free vibrations in three-dimensional homogenous isotropic, visco-thermoelastic hollow sphere subjected to thermally insulated stress free and isothermal boundary conditions. Tokovyy and Ma (2016) proposed a solution for inhomogeneous solid cylinder in axisymmetric thermoelastic problem.

## 1.4 Classical Thermoelasticity

The coupled theory of thermoelasticity arises due to the coupling between thermal and strain fields. Internal and external heat sources are not only responsible for the change in body temperature but also caused by the deformation itself. In classical thermoelasticity, the change in temperature is very small, hence the coupling term in the heat conduction equation and the inertia term in equation of motion are totally neglected. This type of state is known as quasi-static state. When the coupling vanishes, both fields i.e. thermal and strain fields become independent of each other and that type of problem is termed as static. Weiner (1957) and Lesson (1957) explained about coupling effect in detail in their work. Lockett (1958) briefly discussed about the coupled thermoelasticity in surface waves propagation and thermal effects in thermoelastic half space. Boley and Tonlins (1962) studied about transformation of sinusoidal waves w.r.t. time in context to transient temperature and the strain of coupled thermoelastic half space. Windle and Chadwick (1964) discussed the thermally insulated and isothermal boundary conditions on Rayleigh waves for half space elastic solids in heat conduction equation. Nowacki (1968) explained the longitudinal wave propagation of unbounded thermoelastic isotropic medium. Chadwick and Seet (1970) discussed about the effect of thermoelastic wave propagation in heat conducting and non heat conducting transversally isotropic materials. There exists three types of elastic waves called quasi transverse, quasi longitudinal and purely transverse in such types of materials. Only quasi longitudinal and quasi transverse waves are influenced by the coupling effect. Chadwick (1976) explained about the wave propagation of thermoelastic infinitely homogenous (isotropic and anisotropic) heat conducting elastic materials in thermoelasticity. Chadwick derives the series expression for the attenuation coefficients and phase velocities for such types of waves. The classical thermoelasticity model derivation is completely based upon Fourier's law of heat conduction. Landau and Lifshitz (1986) explained the concept of coupled equations of thermoelasticity by using classical thermodynamics methods. Joseph and Preziosi (1989) reviewed that the transmission of heat produced by nearest neighboring waves where the changes in momentum and energy on a microscopic level are propagated along the lines to describe the elastic viscous behavior. Kumar (1989) investigated an axially symmetric hydrostatic tension subjected to the wave propagation of coupled thermoelastic problem in extended elastic plate. Thermally induced wave vibrations of an infinitely solid cavity were discussed by Erbay et al. (1991). Atarashi and Minagawa (1992) explained briefly the transient behavioral changes of sudden heating and cooling effect caused by outer atmosphere in one dimensional thermally insulated multilayered isotropic, homogeneous and thermally insulated composite material. Liangh and Scarton (2002) formulated vibrations in circumferential mode for viscothermoelastic solid as well as thermoelastic solid waves of three dimensional material, fluid filled steel tube in coupled system. Geometrical perfect thermoelastic buckling in circular composite plate which is orthotropic in nature was redefined by Najafizadeh and Eslami (2002).The theory of non-Fourier conduction heat for the calculation of temperature field was investigated by Ning et al. (2000). Ultrasound heat deposition by virtue of laser radiation was given by Achenbach (2005). Gevorgyan (2008) investigated the BVP in coupled thermoelasticity of anisotropic plate as well as strip with the help of asymptotic expansion method. Jovanovic and Sestak (2010) explained the thermoelasticity contact problem approximation with the help of non monotonic function. Belyankova et al. (2012) discussed dynamic harmonic vibration problem in a rigid body surface of a thermoelastic pre-stressed layer. The quasi-static one dimensional problem of thermal stresses, distribution of temperature and displacement in infinite bodies with time dependent sources of heat was investigated by Pawar et al. (2013). Angelov (2013) discussed the analysis of vibrations in steady state rolling medium having thermo mechanical coupled elasticity. The dynamic problem of thermoelasticity with the help of D'Alembert's method was explained by Rossikhin and Shitikova (2014). Su et al. (2020) explained size-dependent elasto-hydrodynamic lubrication line contact of a deformable half-plane rigid cylindrical punch by using density and viscosity of the lubricant vary with the fluid pressure.

## 1.5 Non-Classical Thermoelasticity

In thermoelasticity, there are two types of basic governing equations for linear coupled thermoelasticity, one is wave type of equations of motion also called hyperbolic and another is diffusion type equation of heat conduction called parabolic. In the classical thermoelastic case, it was observed that one part of the energy equation solution approaches towards infinity which means that if any anisotropic material of homogenous elastic medium is dominated to mechanical or thermal disturbances then the effects in displacement and temperature fields are observed at infinite distance from the source of disturbance. Hence there is one part of the solution which has infinite velocity of propagation which is not feasible. Attempts were made by Lord and Shulman (1967) and Meixner (1970) to modify the heat conduction equation based on Fourier law for getting a hyperbolic differential equation for the survival of this problem. Boley (1962) already discussed the existing coupled theory of thermoelastricity in detail. The new theory is termed as generalized theory of thermoelasticity which eliminates all paradox of infinite wave propagation. This theory clarifies the linearly functional relationship of heat flow as well as the temperature gradient. Generalized thermoelasticity also stands for hyperbolic thermoelasticity where the thermo-mechanical load applied to a body is transmitted into a wave like manner. For non-Fourier phenomenon of heat conduction process some generalized thermoelastic models have been developed. Following are the models of generalized thermoelasticity:

(a) Lord-Shulman model (LS)

(b) Green-Lindsay model (GL)

(c) Green-Nigdhi model (GN)

(d) Dual-phase-lag model (DPL)

(e) Three-phase-lag model (TPL)

## 1.5.1 Lord-Shulman (LS) Model

A generalized form of dynamical theory is formulated by Lord and Shulman (1967) by introducing Maxwell-Cattaneo law to generalize Fourier's law of heat conduction by proposing single relaxation time parameter which leads to heat flow. Ackerman et al. (1966) explained that propagation of thermal waves having infinite speed can create frequency vibrations in large amount. Ackerman and Overtone (1969) explained experimentally that the thermal waves are very large, propagating with finite speed besides the range of frequency of thermal excitations, is limited for solid like helium. Lord and Lopez (1970) discussed the propagation of waves in one dimensional thermoelastic solids at low temperature with LS model. Singh and Singh (1972) gave generalized idea about longitudinal waves which are thermoelastic in nature and caused by displacement coupling and temperature fields of finite velocities in unbounded medium. The disturbance which is due to surface point sources in generalized half space was explored by Harinath (1975). Dhaliwal and Singh (1980) gave a detailed description of coupled and generalized thermoelstic problems in continuum mechanics. Basu (1982) solved basic field equations in the propagation of waves for infinite unbounded homogenous spherical cavity in the reference of generalized thermoelasticity. Mondal (1983) gave an idea about propagation of plane wave in thermoelastic solid kept at consistent temperature absorbed in infinite liquid to symmetric and skew symmetric vibrations. By using Laplace transformation technique Sherief (1986) solved the system of coupled equations and presented the results for temperature and stress distribution in an infinite elastic material in generalized thermoelasticity. Sherief and Anwar (1994) presented traction free two dimensional solution of thick plate in which upper and lower surfaces are considered axisymmetric temperature heat supply in context of LS model of generalized thermoelasticity. Norris and Photiadis (2005) derived the result of thermoelastic relaxation structures, which enables thermoelastic damping in vibrating elastic solids. The interactions in thermoelastic half space comprises of ramp type heating by virtue of finite element method was given by Ibrahim (2012). Later on Sarkar (2013) discussed the generalized thermoelastic problem with one relaxation time parameter which includes heat source in a vector-matrix equation using Laplace transformation.

The theory of generalized elastic diffusion in half space was given by Allam (2014). Kumar and Devi (2016) investigated problem of axisymmetric thick circular plate in couple stressed theory with mass diffusion by using Laplace transforms technique in the context of LS model. Kiani and Eslami (2017) explained the analysis of an isotropic homogeneous layer with the help of LS theory of generalized thermoelasticty where the relaxation time and coupling parameters are very large in two coupled partial differential equations i.e. equation of motion and energy equations. Heydarpur and Malekzadeh (2018) studied the thermoelastic waves in multilayered spherical shells having functionally graded layers in thermal boundary conditions by applying LS model under the effect of finite heat wave speed. Oskouie et al. (2019) and Bouslimi et al. (2021) studied the LS theory with reference to the nonlinear coupled thermo-viscoelasticity of a layer using Kelvin-Voigt theory of viscoelasticity to analyze the influence of nonlinearity in the energy equation. Hayati et al. (2019) studied four new scalar potential functions which was introduced for uncoupling of coupled displacement–temperature equations satisfying uncoupled partial differential equations having lesser complexity than the coupled displacement–temperature equations. Wei et al. (2020) explored the Lord-Shulman thermoelasticity model which describe the wave dissipation due to effect of fluid and heat in homogeneous media as the displacements-temperature solution . Alharbi (2021) presents a new model of volume fraction in photothermal effect based on LS theory in generalized thermoelastic medium by imposing normal mode analysis.

## 1.5.2 Green-Lindsay (GL) Model

Green and Lindsay (GL) (1972) used the generalized theory of coupled thermoelasticity to prove that there exist second sound effects. This theory was linearized and was based on the modified constitutive equations for entropy production inequalities given by Green and Laws (1972). Green (1972) explained that GL theory consists of stress and entropy constitutive relations, generalized by introducing different types of two relaxation time parameters in inequalities. Later on Green and Naghdi (1977) discussed the uniqueness of constitutive relations and propagation of waves which were given earlier by Green and Lindsay (1972). The formulation problem of dynamical disturbances in one dimensional thermoelastic half space due to strain and temperature on the plane boundary was explained by Chandrashekhariah (1981). Chandrashekhariah and Srikantiah (1984) discussed briefly the propagation of plane waves comprises in homogenous isotropic medium with unbounded thermoelastic solid rotated uniformly with angular velocity. Again Chandrashekhariah and Srikantiah (1985) investigated the propagation of waves on the edges of a flat thin plate having infinite length by using time rate dependent theory. Tamma and Namburu (1992) presented detailed ideas about effectiveness of finite element analysis modeling approach due to second sound effects in thermoelastic materials. Sherief (1994) discussed half space thermoelasticity problem with two relaxation time parameters having thermal as well as mechanical shock by using Laplace transformation technique. Chandrashekhariah (1999) discussed the problems related to one dimensional disturbances of the half space arises due to thermal impulse on the boundaries with the help of GL and LS theories. Strunin (2001) explore the theory of entropy production inequality in which no limitation on relaxation time parameter arises is considered as the modified GL model. Othman et al. (2002) investigated 2D thermo-viscoelasticity theory having two relaxation times to get the accurate expression for thermal stresses, temperature distribution and the displacement components with normal mode analysis. Othman (2004) derived an expression for the effect of rotation on the propagation of plane waves with two relaxation times in generalized thermoelastic solids. For an infinitely long isotropic cylinder having temperature dependent properties Mukhopadhyay and Kumar (2009) solved a problem with one time relaxation in generalized thermoelasticity. Fatimah (2012) investigated the thermoelasticity problem with two relaxation time parameters of longitudinal vibrations in an infinite circular cylinder. Wang et al. (2013) formulated a new generalized thermoelastic solution for an isotropic elastic medium having temperature dependent conditions. Othman and Atwa (2014) studied the effect of magnetic field in two dimensional problems of fiber-reinforced thermoelastic materials by presenting CT, LS and GL theories. Avalishvili et al. (2017) proved the existence and uniqueness of solution of the GL non classical model of inhomogeneous anisotropic thermoelastic materials having two relaxation times on space variables. Jun and Xue (2018) explored the problem of thermoelastic materials with temperature rate and transient responses strain rate with traction free at one end, subjected to temperature rise in GL thermoelastic model by using the Laplace transform method. Aouadi et al. (2019) proved the existence and stability of a visco-thermoelastic multidimensional contact problem of displacement and temperature in viscoelastic solid by using Kelvin–Voigt law in context with GL theory of thermoelasticity. Sarkar and Mondal (2020) explained the propagation and reflection of thermoelastic plane waves in the stress-free as well as thermally insulated surfaces of homogeneous, isotropic elastic half-space in context to the modified GL theory of generalized thermoelasticity with two-temperatures. Mondal and Pal (2021) studied the thermal shock effect on one end in one-dimensional piezoelectric material by using electric potential, displacement, stress and temperature based on the GL model of generalized thermoelasticity.

## 1.5.3 Green-Naghdi (GN) Model

On the basis of entropy production inequality, the equation of balance energy was formulated for the constitutive equations of thermoelasticity. Green and Naghdi (1977, 1991 and 1992) developed the generalized thermoelasticity theories with finite speed in the propagation of thermoelastic disturbances. The thermoelasticity theory without energy dissipation was given by Green and Naghdi (1993) is known as Green-Naghdi (GN) theory. It is that mathematical model which is described by the system of field equations, where rate of heat flux and temperature gradient relations are compared with classical coupled theory of thermoelasticity, replacing the Fourier's law of heat conduction. As the equation of heat conduction does not contain temperature change, therefore representing an un-damped thermoelastic waves referred as GN theory of thermoelasticity without energy dissipation (TWED). They derived the complete set of constitutive governing equations for homogenous isotropic materials with temperature and displacement fields in linearized theory of thermoelsticity. They formed three types of models in thermoelasticity for homogenous and isotropic materials. The first model is similar to coupled thermoelastic model, while the other two models admitting second sound effects. Linearized version of this model has an important feature that it does not contain dissipation of thermal energy. Green and Naghdi (1992) explained that the response function of thermoelastic materials behavior have more general features as compared to the classical thermoelasticity. Green and Naghdi (1993) also represented thermoelasticity theory by introducing thermal relaxation times based on basis of the balance of entropy law and balance of energy law. Chandrashekhariah (1998b) explained two types of variational principles in the context of generalized thermoelasticity theory without energy dissipation for homogenous isotropic materials. One is variational principle of Boit-Hamilton type and other is principle of

Betti-Rayleigh type. Hossen and Mallet (2000) presented the same theory to study the effect of thermoelastic wave propagation in a plane having punched hole. Thermoelastic thick plate behavior under lateral load was investigated by Sharma and Sharma (2004) by using finite element method and Laplace transformation technique. Othman and Kumar (2009) formulated problem of reflection of the thermoelastic-magneto waves due to temperature dependent effect in generalized thermoelastic materials. Atwa (2014) proposed magneto-thermoelastic wave interactions in homogenous stressed isotropic elastic half-space affected by two temperatures using mathematical methods in lieu of GN theory of type II and type III. Othman and Atwa (2014) also studied GN model and concluded that the effect of reinforcement on total deformation of an infinite space is weakened by thermal shocks. Othman and Tantawi (2016) investigated the effect of the gravitational field of 2D thermoelasticity under GN theory with and without energy dissipation of thermal loading of a laser pulse. Ezzat et al. (2017a) developed a mathematical model for thermo- viscoelastic solids by applying Fourier law with memory-dependent derivative. Ameen et al. (2018) examined a thin slim magnetic strip problem subjected to a moving heat source by assuming the governing equations with theory of fractional order and viscous parameter. Aldawody et al. (2019) studied the thermo-mechanics theory for the propagation of thermal waves with finite speed and established a linear theory of thermoelectric fluid having fractional order of heat transfer. Ahmed and Abouelregal (2020) presented a modified model of heat conduction which include higher order time derivative derived by extending GN theory without energy dissipation, which describes the thermoelastic waves in a homogeneous isotropic perfect conducting unbounded solid material containing a spherical cavity by using the Laplace transform technique.

## 1.5.4 Dual-Phase-Lag (DPL) Model

In dual-phase-lag (DPL) model, there are two types of different time translations. One is a phase lag of heat flux while other is phase lag of temperature gradient. This model is generally known as Chandrashekharaiah-Tzou (C-T) model. This model of generalized thermoelasticity was proposed by Chandrashekharaiah (1998a) and Tzou (1995), which was an extension to the classical thermoelastic model of the thermoelasticity, where Fourier's law of heat conduction was replaced by approximation to modified Fourier's law having two different types of translations.

Modified Fourier's law of Taylor series approximation along with field equations results to complete system of equations which describe DPL model .This model predicts the thermoelastic disturbances in the wave like manner subjected to the condition if the approximation is linear phase lag of heat flux or quadratic in phase lag of heat flux and linear in phase lag of temperature gradient. Lofty (2016) studied isotropic thermoelastic medium with internal heat source moving with constant speed by introducing harmonic wave analysis in the reference of DPL model of generalized thermoelasticity. The resulting vibrations are obtained under the effects of parameters based on dual phase lag model. Mittal and Kulkarni (2016) studied DPL model using the fractional theory of thermoelasticity with relaxation time by using generalized Fourier law of heat conduction. Ezzat et al. (2017b) developed a mathematical model for thermoelastic materials with memory-dependent derivatives for DPL model in context to heat conduction law. Biswas and Sarkar (2018) investigated the plane waves in the porous thermoelastic medium in the reference of DPL model of generalized thermoelastic materials. They examined that the three longitudinal waves, elastic, thermal and volume fraction, got decoupled from rest of the motion and not caused by the thermal and volume fraction fields. Mondal et al. (2019) developed a generalized thermoelasticity mathematical model by introducing Laplace transform to explore the transient phenomena for a piezoelastic half-space with magnetic field in reference of DPL model of generalized thermoelasticity. Sharma et al. (2021b) also analyze the vibrations of a thermoelastic hollow sphere with voids under DPL model and Eringen's nonlocal elasticity by imposing the stress-free thermal boundary conditions. Kumar et al. (2021) examined the problem of reflection of plane harmonic waves in a micropolar nonlocal thermoelastic medium with void pores in lieu of DPL and LS models. They show that a coupled longitudinal displacement wave is made incident when the reflection coefficients in various reflected waves and their energy ratios are computed analytically. Nonlocal and micropolar parameters on the variations of energy ratios and reflection coefficients of various reflected waves are influenced.

### 1.5.5 Three-Phase-Lag (TPL)

A three-phase-lag (TPL) model has been comprised of the theory of generalized thermoelasticity which was formulated by considering that heat conduction law includes thermal displacement gradient and temperature gradient among the constitutive variables. The modified Fourier law is replaced by an approximation with three different relaxation time parameters, the thermal displacement gradient, the heat flux gradient and the temperature gradient. Thus the model formulated is just an extension of the thermoelastic models given by Lord–Shulman (1967), Green–Naghdi (1993), Chandrashekharaiah (1998a) and Tzou (1995). The new model was based upon TPL theory of generalized thermoelasticity, introduced by Roychoudhuri (2007). In this model, Roychoudhuri introduced three different phase–lags, named as $\tau_q$ (phase-lag of the heat flux), $\tau_T$ (phase-lag of the temperature gradient) and $\tau_\upsilon$ (phase-lag of thermal displacement), which obey classical Fourier's law: $\vec{q} = -K\vec{\nabla}T$ as $\vec{q}(P,t+\tau_q) = -\left[K\vec{\nabla}T(P,t+\tau_T) + K^*\vec{\nabla}\upsilon(P,t+\tau_\upsilon)\right]$. The TPL model is useful in various applications related to pure phonon scattering, electron-phonon interactions and nuclear boiling etc. The uniqueness of the TPL model of generalized thermoelasticity was explained by Quintanilla (2009). Kar and Kanoria (2009) proposed TPL model of spherical shell containing the viscoelastic solid due to small input of the temperature in reference of generalized thermoelasticity. Keles and Tutuncu (2011) used Laplace transformation technique to find the vibration analysis of functionally graded elastic cylinder and sphere which have been presented for free and forced vibrations. D'Apice et al. (2016) analyzed the time differential TPL model of coupled thermoelasticity with the basic initial BVP of the Lagrange type model to establish an identity. Mondal et al. (2017) investigated the ramp type heat effect of thermoelastic materials having gold nano-beam resonators by inducing two temperature theory in the reference of TPL model of generalized thermoelasticity. Magana et al. (2018) discussed about the stability in TPL model of heat conduction in context of two temperature theory of thermoelastic solids. Kaur et al. (2020) analyze the problem of refraction and reflection of plane wave at the boundary of two different transversely isotropic thermoelastic solid half-space with two temperature with TPL model of memory dependent derivative. Mondal and Kanoria (2020) explained the solution of thermal distributions in moving thin slim rod having memory dependent thermoelasticity in the context of TPL theory. Sharma et al. (2020a, 2020b) investigated free vibrations of thermoelastic cylinder as well as sphere with voids material in the context of generalized theories of thermoelasticity. Alharbi (2021) study the effect of inclined load and two temperature on a micropolar thermoelastic medium with voids by applying normal mode technique to analyse

TPL model. Sharma and Thakur (2021) extended Eringen's nonlocal thermoelasticity model for the analysis of free vibrations of thermoelastic sphere with voids under TPL model in lieu of traction-free thermal boundary conditions. Sharma et al. (2021) explore the effects of TPL model to analyze three-dimensional free vibrations in viscothermoelastic solid cylinder by applying TPL parameters.

## 1.6 Voids

The voids presence affects the estimation of mechanical and physical properties of composite material. We have supposed that the voids contain nothing of mechanical but energetic significant and the distribution of voids is partially to the medium. Voids can act as crack initiation sites as well as allow moisture to enter in the composite and contribute to the anisotropy of the composite materials. Voids content in composite material is represented as a ratio, also called voids ratio, where the volume of voids, solid material and bulk volume are taken into account. The voids in soil mass represent the pores between the soil particles including the volume of water and the volume of air. Both volume of water and volume of air add up together is said to be volume of void in the soil $V_v - V_a - V_w = 0$.Voids ratio can be calculated by the formula $V_v = e V_t$, where $e$ is the voids ratio of the composite, $V_v$ is the volume of the voids and $V_t$ is the volume of the bulk material. Linear theory of elastic materials with voids is one of the most significant generalizations of the classical theory of elasticity. This theory has useful utility for investigating various types of geological and biological materials for which elastic theory is insufficient. This theory is concerned with elastic materials consisting of distribution of small pores (voids), in which the voids volume is included among the kinematics variables and in the limiting case of volume tending to zero, the theory reduces to the classical theory of elasticity.

### 1.6.1 Voids Elasticity

The theory of solid voids elastic materials has been extended from classical theory of elasticity. The voids theory refers to the distribution of pores (spaces) in elastic materials composed of kinetic variables, which means that there is no significance of energetic or mechanical properties. The theory of voids in non-linear version was proposed by Nunziato and Cowin (1979). Four years later, Cowin and

Nunziato (1983) formulated the linear version of this solid voids theory. This theory differs from classical linear elasticity theory as the volume fraction corresponds to the void volume is taken as an independent variable. Puri and Cowin (1985) explained about the harmonic plane waves in linear elasticity in two dilatational waves with voids material. Iesan (1985) investigated some theories in detail based on the properties of elastic as well as thermoelastic materials with voids. That theory has added some new features on porous (void) materials, which allows a void (porous) body to reduce and expand overall volume in the absence of body forces. The theory of elastic voids material has many applications in the field of science and engineering such as petroleum industry, biology, geology, material science and also in the study of artificial porous (void) materials. Chandrasekharaiah (1987) explained the effect of elastic surface waves on Rayleigh wave with voids in lieu of continuum mechanics. Marin (1997) investigated the BVP based on thermoelastic solid with voids material to prove the uniqueness of the solution. Tomar (2005) obtained frequency equations for propagation of waves in a plate of micropolar elastic material with voids. Sharma et al. (2008) discussed the vibrations of three dimensional cylindrical penal with voids and presented the field functions analytically. Tomar and Ogden (2014) studied the 2D propagation of an elastic isotropic solid with voids having constant angular velocity. Singh et al. (2017) studied the propagation of harmonic plane waves in infinite nonlocal elastic solids with voids. Kaur et al. (2018) studied the surface wave propagation in nonlocal homogeneous isotropic elastic material half space with voids. Kaur et al. (2019) explained the transmission of Love-type nonlocal elastic waves underlying to the half-space solid with voids. Khurana and Tomar (2019) investigated the reflection and transmission of plane waves by unraveling two different nonlocal micropolar elastic materials in perfect contact. Kaur et al. (2021) presented the vibration analysis of elastic waves with voids having uniform thickness in nonlocal elastic medium by using boundary conditions. Kumar and Tomar (2022) examined one-dimensional wave propagation during displacement of single slip plane and direction in linear isothermal elastic and plastic anisotropic material with voids.

## 1.6.2 Voids Thermoelasticity

Love (1944) discussed the problems in the reference of non classical and classical theories of thermoelasticity. Boit (1956) explained about the concept of the generalized free energy which was developed for thermoelasticity. That was a unified treatment as a result of which some basic physical laws were adopted by all irreversible phenomenona and bring out hidden relationships with other phenomenon which relates a particular field of thermoelasticity in a broader context. Thermoelasticity comprises of a large category of phenomena including the general heat conduction theory, thermal strains and stresses. The reverse effect of temperature distribution in elastic bodies arises due to the elastic deformation itself result thermoelastic dissipation. Dhaliwal and Singh (1980) explained in detail about the coupled thermo elasticity. Singh (2011) examined the analysis of thermoelastic solid with voids and diffusion in the reference of the Lord-Shulman model of generalized thermoelasticity. Bachher et al. (2014) investigated the generalized thermoelastic GL model of solid voids materials which were subjected to thermal heat sources applied on a plane area. Tomar et al. (2014) studied the harmonic wave's transmission in an infinite thermo-viscoelastic solid with voids. Four types of waves were found, in which, one was a shear wave and the other three were dilatational waves. The propagation speed of all waves found to be frequency dependent as well as complex valued. Reflection phenomena with the help of boundary conditions for thermo-viscoelastic half-space with voids have been explained. Usal et al. (2017) explained the behavior of thermo-electro-mechanical structures of thermoelastic dielectric materials in the preview of continuum damage mechanics. Sarkar and Tomar (2019) presented the plane wave propagation in time harmonics of nonlocal thermoelastic solid with voids. These waves are dispersive in nature and the coupled dilatational waves and transverse waves are respectively attenuating and non-attenuating. Mitali and Sarkar (2019) established a new theory for non local generalized thermoelastic materials with voids. Singh and Singla (2019) investigated the effects of diffusion, rotation and voids on plane waves in thermoelastic solids by LS model in rotating thermoelastic solid with voids. Vidler et al. (2021) studied the third order expansion of a strain energy density function as well as finite strain elastic theory to get effective linear and nonlinear properties of a material having diluted distribution of voids.

## 1.7 Nonlocal Theory of Elasticity

The theory of nonlocal elasticity states that applied stress of continuous body at a point '$x$' within the body is considered as the functions of strain at that point as well as the strain at all other points of the continuous body**.** This view is in accordance with atomic theory of lattice dynamics and the experimental observations on phonon dispersion. Crystal defects may be modeled mathematically as nonlocal elastic solid with voids. Edelen and Laws (1971) thoroughly explained the nonlocal elasticity theory for the equations of motion and the constitutive relations with suitable thermodynamic restrictions. They developed the constitutive relations and equation of motion under appropriate thermodynamic constraints. The characteristics of nonlocal theories are perfectly suited to the materials which have equivalent internal and external characteristic lengths. The nonlocal elasticity was further extended broadly by McCay and Narsimhan (1981). Altan (1984) and Craciun (1996) studied the theory of nonlocal elasticity in the context of uniqueness of the theory of thermoelasticity. Rayleigh waves in rotating nonlocal magnetoelastic half-plane were studied by Roy et al. (2015). Wave propagation in nonlocal microstretch solid was investigated by Khurana and Tomar (2016). Bachher and Sarkar (2019) studied the nonlocal theory of thermoelastic materials with voids in the context of fractional derivative heat transfer. Mondal et al. (2019) described thermoelastic waves in DPL model of thermoelastic materials with voids based on the nonlocal theory of elasticity. Sharma et al. (2020c) explored the free vibration analysis of functionally graded nonlocal thermoelastic spheres with LS and DPL models in the reference of generalized theories of thermoelasticity. Kaplunov et al. (2022) study the proportional analysis of integral and differential formulations for BVP in nonlocal elasticity. The most general type of representation of the fundamental relationship in nonlocal elasticity involves an integral over the entire domain of interest. In Eringen's nonlocal model, constitutive relations for linear, homogeneous and isotropic nonlocal elastic solid without memory are as follows:

$$t_{ij}(x) - \int_V G\left(|x' - x|, \xi\right) t_{ij}^L(x') dV(x') = 0. \tag{1.7.1}$$

The terminology of $t_{ij}^L$ and $e_{ij}$ are given as

$$t_{ij}^L(x) = \lambda \delta_{ij}\, e_{kk}(x') + 2\mu e_{ij}(x'), \tag{1.7.2}$$

$$e_{ij}(x') = \frac{1}{2}\left(\frac{\partial u_i(x')}{\partial x_j'} + \frac{\partial u_j(x')}{\partial x_j'}\right). \tag{1.7.3}$$

Equation (1.7.1) relates that the universal stress tensor $\mathrm{t}_{\mathrm{ij}}$ defined at the point $x$ to the local stress tensor $\mathrm{t}_{\mathrm{ij}}^{\mathrm{L}}$ defined at all points $x'$ in the volume $\mathrm{V}$ by using nonlocal kernel $G$. The quantity $\xi$ is the ratio of the internal characteristic length to the external characteristic length. Three types of non-locality which are commonly known in the literature are defined below.

## 1.7.1 Types of Non-Locality

A physical quantity the effect $e$ at point $x$ in space and time $t$ denoted by $\left(e\ (x_1, x_2, x_3, t)\right)$ is locally dependent on some other physical quantity that is the cause at

the same point $x$ at the same time t and marked by $\left(c\ (x_1, x_2, x_3, t)\right)$ has the general form

$$e\ \left(x_1, x_2, x_3, t\right) - \ e\ \left(c(x_1, x_2, x_3, t)\right) = 0. \tag{1.7.4}$$

## (i) Spatial Non-Locality

Spatial non-locality is one in which the effect E at a given place $x$ in time $t$ is determined by the causes at all points at the same time, expressed below as:

$$\mathrm{E}\,(x_1, x_2, x_3, t) - \int_{\mathrm{V}} \bar{\sigma}\ \left(\left|x - \bar{x}\right|, \xi\right)\ \mathrm{e}\left(\mathrm{c}\,(x_1', x_2', x_3', t)\right) dx_1'\, dx_2' dx_3' \ = 0\,, \tag{1.7.5}$$

where $\xi$ is the ratio of internal characteristic length i.e. the size of the grain or lattice parameter to the external characteristic length which is the size of the sample or wavelength in the volume V of the body.

## (ii) Temporal Non-Locality

In temporal non-locality, the effect E at a given point $x$ in time $t$ is determined by the history of causes at that point $x$ in time over all previous times and at the present instant. This shall be expressed as:

$$E\,(x_1, x_2, x_3, t) - \int_{-\infty}^{t} \varpi\left(\left|t - t'\right|, Q\right) e\left(c\,(x_1, x_2, x_3, t)\right)\ d\,t' = 0\ , \tag{1.7.6}$$

where $Q$ is the ratio of internal characteristic time i.e. the time it takes for a signal to move between molecules (also known as relaxation time) to the external characteristic time which is the time of application of external action or period of vibration.

## (iii) Mixed Non-Locality

For this type of nonlocality, the effect E at the point $x$ at time $t$ depends on the causes at all points $x'$ and at all times $t' \leq t$, expressed as:

$$E(x_1, x_2, x_3, t) - \int_V \int_{-\infty}^{t} \bar{\gamma}\left(|x - \bar{x}|, t - t', \xi, Q\right) e \left(c(x_1', x_2', x_3', t)\right) dt'\, dx_1'\, dx_2'\, dx_3' = 0 \tag{1.7.7}$$

In (1.7.5)-(1.7.7), the functions $\bar{\sigma}, \varpi$ and $\bar{\gamma}$ are the nonlocal kernels which include parameters that correspond to the nonlocal elasticity.

## 1.8 Nonlocal Thermoelasticity

Yu et al. (2015) suggested size-dependent generalized thermoelasticity based on Eringen's nonlocal elasticity model. They also used this new theory to investigate the temporary response of thermoelastic nonlocal infinite medium with voids in one dimension. Yu et al. (2016) also suggested nonlocal thermoelasticity which was based on nonlocal elasticity and nonlocal heat conduction. Jun et al. (2016) discussed the nonlocal thermoelasticity based on nonlocal heat conduction and nonlocal elasticity. Bachher and Sarkar (2019) investigated the transient response of a thermoelastic nonlocal infinite medium with voids in one dimension. Sarkar and Tomar (2019) suggested plane wave propagation in nonlocal thermoelastic voids solid along with thermal relaxation time. Mondal et al. (2019) also explained in detail the propagation of waves in dual phase lag thermoelastic materials with voids in lieu of Eringen's nonlocal elasticity theory. In recent times, Sharma et al. (2020d) derived the relations as well as equations for the nonlocal thermoelastic solids with diffusion. Luo et al. (2021) studied model of nonlocal thermoelastic materials which is developed to envisage the behavior of nanostructures in excessive environments. Saeed and Abbas (2022) presented a nonlocal thermoelastic model without energy dissipations in the context of nanoscale material by applying Eigen-value approach.

## 1.9 Maxwell Equations

Maxwell's equations are named after scientist James Clerk Maxwell who put the equations together in 19th-century. Maxwell's equations are condensed, well-defined and attractive system of equations. Those equations express the performance of electric fields, magnetic fields, and their condensed relationships. The well defined field of electricity and magnetism lead to this minster of equations, alliance of varied phenomenon and a secret of the universe. The importance of Maxwell's equations cannot be easily understated, inside the dense model of equations, all modern electronic, electrical and magnetic, radio as well as the power generation systems occur. Maxwell’s equations are the basic types of equations on which the structure of electromagnetism can be formed or built. These are four equations which comprise to form Maxwell’s equations. Gauss’ law describes the electric field, while the the magnetic field as well as the non-existence of magnetic monopoles is described by Gauss’s law for magnetism. Ampere’s law shows that up to what extent a current can be induced by a moving generator or that a magnetic field is capable of being induced by a current. The statements of these four equations are given below as:

(1) Electric field diverges from the electric charge, which is an expression of the Coulomb force.

(2) Coulomb’s force acts between the poles of a magnet besides that there are no isolated magnetic poles.

(3) Changing magnetic fields produces electric fields, which is an expression of Faraday’s law of induction.

(4) Changing electric fields and electric currents produces circulating magnetic fields.

In the given below expressions, the Greek letter rho $\rho$ is the charge density, $J$ is the current density, $E$ represents the electric field, and $B$ represents the magnetic field. $D$ and $H$ are the field quantities which are proportional to $E$ and $B$ respectively. These four Maxwell equations are as:

$$div\ D = \rho \tag{1.9.1}$$

$$div\ B = 0 \tag{1.9.2}$$

$$curl\ E = -\frac{dB}{dt} \tag{1.9.3}$$

$$curl\ H = \frac{dD}{dt} + J \tag{1.9.4}$$

The Coulomb's law of electrostatics had been established in 1780’s era. Ampere’s law was published in 1825 by Ampere. Michael Faraday revealed the electromagnetic induction by doing experiments and he theoretically emphasized on the lines of forces in the electromagnetic induction. The problem of direction of induction solved in 1834 by Lenz and Neumann gave that equation by which one can calculate the induced force due to alter of magnetic flux. James C. Maxwell published a series of papers from 1850-1870. In the 1850s, Maxwell was overwhelmed by the concept of Faraday's lines of forces while he was working at University of Cambridge . Maxwell (1856) published his paper in electromagnetism on Faraday's lines of forces by means of the analogy of incompressible fluid flow for reproduction the magnetic lines of forces. Maxwell (1861a, 1861b, 1861c, 1862a) published four papers series on physical lines of forces, where he used the mechanical models to create the electromagnetic field. He also investigated the vacuum as a kind of insulating elastic medium for the accountness of the stress of the magnetic lines of force which was given by Faraday. Maxwell (1862b) published a paper to derive the speed of light (*c*) from the velocity expression of electromagnetic wave in context to vacuum constants. Maxwell (1865) published a dynamical theory of the electromagnetic field in mathematical forms, which are known as Maxwell's equations.

## 1.10 Electro-Magneto Thermoelasticity

Electromagnetism is that branch of science which deals with charge, forces and fields associated with charge. Electricity and magnetism are two different aspects of electromagnetism. Eringen (2002) proposed theories for nonlocal elasticity, nonlocal thermoelasticity and nonlocal electro-magneto elastic materials which depends upon the different methods of additional functions of continuum mechanics and explained the stress-strain relationships, governing equations and laws of equilibrium. Ezzat and Youssef (2005) investigated a mathematical model which based on the conducting media in magneto-thermoelasticity in preview of generalized theories of thermoelasticity.. Allam et al. (2010) discussed the analysis of generalized GL model of electro-magneto thermoelastic spherical cavity of infinite perfectly conducting material. Das et al. (2013) explore finite element method in magneto thermoelastic cylinder with TPL model of generalized thermoelastity in the presence of heat sources. Othman et al. (2016) studied the modified Ohm's law with micro-temperatures and rotation in magneto-thermoelastic material. The rotation effect in magneto-thermoelasticity with voids material was explained by Othman and Hilal (2017). Mondal (2019) explored the effect of DPL model in magneto thermoelastic rod with moving heat source in nonlocal elasticity theory. The moving heat sources in thin slim rod with the effect of TPL model of generalized magneto thermoelasticity was investigated by Mondal and Kanoria (2020). Said and Othman (2021) studied the transmission of electro-magneto thermoelastic disturbances which were produced by the thermal shock in conducting elastic half-space.

## 1.11 Vibrations of Spherical and Cylindrical Structures

Lamb (1882) investigated two types of vibrations, one was first class free vibrations having zero radial displacement and volume change while other was the second class free vibrations having zero radial components in curl of displacement vector in an isotropic sphere. Chadwick (1962) obtained solution for the coupled linear thermoelastic equations with oscillatory motion in an elastic body. Chau (1994) investigated the exact frequency equations for vibrations of transversely isotropic cylinder having finite length. Chau (1998) derived the exact frequency equations of isotropic elastic sphere for toroidal mode of vibrations. Sharma (2001) examined the analysis of free vibrations of homogeneous transversely isotropic cylindrical panel which was based on three dimensional thermoelasticity. Chen and Ding (2001) explored three-dimensional free vibrations analysis for spherically isotropic multi-layered hollow sphere. The vibrations of transversely isotropic solid spheres by finite element method technique were studied by Buchanan and Ramirez'a (2002). Neyfeh and Arafat (2006) investigated the axisymmetric surface analysis of closed spherical shell with the help of variational approach. Chen et al. (2008) solved the problem of transversely isotropic cylindrical shell for the free vibration with modified Bessel function solution. Sharma and Sharma (2010) explained 3D vibration analyses of the thermoelastic solid sphere in presence of thermally insulated and isothermal traction free boundary conditions. Fazelzadeh and Ghavanloo (2012) examined the axisymmetric vibrations of a fluid-filled spherical shell in the context of nonlocal elasticity theory. Ghavanloo and Fazelzadeh (2013) presented the radial vibration analysis of elastic, homogeneous and isotropic nanoscale spherical shells in lieu of the nonlocal elasticity theory. Sharma et al. (2013) studied the exact vibrational analysis of homogenous viscothermoelastic rigidly fixed hollow sphere. Sharma et al. (2014) focused on the analysis of free vibrations of axisymmetric functionally graded hollow spheres in preview of radial direction with simple power law. Sharma et al. (2016) obtained the analytical and computational solutions for transversely isotropic FGM material thermoelastic hollow cylinder with heat source in the context of generalized thermoelasticity. Sherief and Allam (2017) investigated a two dimensional problem of electro–magneto interaction of an infinite long solid circular cylinder. Sharma and Mishra (2017) developed a model which concerned with the free vibrations of thermoelastic, isotropic, FGM and transverselly isotropic sphere in the context of generalized thermoelasticity theory with one relaxation time parameter. The wave propagation of nonlocal thermoelastic harmonic plane waves in a thermoelastic medium in context of GN theory of generalized thermoelasticity was investigated by Sarkar et al. (2019). Sharma et al. (2020e) studied the vibration analysis in isotropic nonlocal thermoelastic cylinder with voids using time-harmonic technique. Sharma and Thakur (2021) studied free vibrational analysis of generalized thermoelastic sphere with voids under TPL model in context of Eringen’s nonlocal elasticity. Sharma et al. (2021c) examined the vibrations of rigidly fixed electro-magneto nonlocal elastic voids cylinder with generalized thermoelasticity in isothermal and thermally insulated boundary conditions. Sharma et al. (2022a) discussed the free vibrational analysis of transversely isotropic nonlocal electro-magneto thermoelastic hollow cylinder with voids material in the preview of the generalized thermoelasticity. Sharma et al. (2022b) explored free vibrations of electro-magneto transversely isotropic elastic nonlocal hollow sphere with voids in the context of DPL model of generalized thermoelasticity in radial direction.

## 1.12 Motivation and Objectives

In order to fill the research gaps in literature and keeping in view the applications of transversely isotropic thermoelastic material in various engineering and physical industries, the following objectives of the present study have been laid:

1. To study free vibrations of nonlocal thermoelastic cylindrical structures with voids
2. under the impact of magnetic field.
3. To study Dual Phase Lag model for transversely isotropic nonlocal thermoelastic spherical disk structures with voids under the impact of magnetic field.
4. To study Three Phase Lag model for transversely isotropic nonlocal thermoelastic cylindrical disk with voids under the impact of magnetic field.
5. To formulate and solve mathematical models related to such materials.
6. To compare results with existing results for materials.

7. To develop mathematical models which are useful for applications in mathematics and industries where cylindrical/Spherical structures are in frequent use.

The present thesis is the study of vibration analysis of transversely isotropic, thermoelastic cylinder, hollow cylinder, spherical curved plate and local, non-local spheres and cylinder with voids under the impact of Magnetic field, subjected to two different types of boundary conditions namely:

(i) Stress free, thermally insulated.
(ii) Stress free, isothermal conditions.

The problems have been modeled with lend a hand of non-classical theories of thermoelasticity developed by Lord and Shulman (1967) and Green and Lindsay (1972) subjected to elastic materials under different conditions. The generating equations have been solved by using computational tools MATLAB. The computer replicated results in reverence of frequency, frequency shift, thermoelastic damping, dissipation factor, temperature change, displacement components and stresses have been presented graphically to show the behavior of the model as well as of materials. The mathematical elasticity theory provides a rich skeleton to the study of such types of applications. This may also offer many exciting and demanding mathematical problems in its own right, relating to the governing PDE and qualitative properties of their solutions. Here we use time harmonic variation technique to transforming the governing equations and constitutive relations into ordinary differential technique, Iteration method to create numerical data with the help of computational tool MATLAB. Systems of linear equations have been solved by means of matrices method and crammers rule. Numerical data have been computed graphically with the help of computational tools.

## 1.13 Contributions of the Present Work

Present work consists of 07 chapters:

## Chapter-01

The Chapter is introductory in nature and concise the necessary results that are relevant to the present thesis. This introductory chapter lays down the fundamental framework, study of basic theories which helps in solving various mathematical models presented in subsequent chapters. A brief account of the motivating factors, objectives and literature survey for various structures are presented herein.

## Chapter-02

In this chapter the free vibrations of transversely isotropic nonlocal electro-magneto thermoelastic hollow cylinder with voids are addressed in the preview of generalized thermoelasticity. The governing equations and the constitutive relations are transformed into coupled ordinary differential equations by applying time harmonic variations. The boundary conditions of the outer and the inner surfaces of the hollow cylinder are considered to be traction free, no change in voids volume fraction and thermally insulated/isothermal temperature field. The analytical results for frequency equations are presented and validated with existing literature. To explore the free vibration analysis from the considered boundary conditions, the numerical iteration method are applied to create data by using MATLAB software tool. The obtained analytical results are represented graphically with the assistance of numerical computations and simulations in absence/presence of magnetic field for nonlocal/local thermoelastic materials.

## Chapter-03

The solid voids theory with elastic materials is extended from classical theory of elasticity. The voids theory is the distribution of pores in elastic materials comprised in kinetic variables and considered as there is no significance of energetic or mechanical properties. Herein, the main aim of the current chapter is to present DPL model of transversely isotropic generalized electro-magneto nonlocal thermoelastic hollow sphere/disk with voids material. The time harmonic variations have been employed to constitutive relations and governing equations. The elimination method has been employed to find field functions to present analytical results and numerical Iteration method has been applied to assumed boundary conditions. To check the effects of DPL model and nonlocal elasticity, the analytical results for frequencies and thermoelastic damping in absence/presence of magnetic field, have been represented graphically.

## Chapter-04

In this chapter the stress–strain–temperature relations, strain–displacement relations and governing equations are shown for electro-magneto transversely isotropic nonlocal elastic hollow cylinder with voids in the reference of three-phase-lag effect of heat conduction. The simultaneous differential equations are eliminated by applying elimination technique to obtain unknown field functions such as dilatation, equilibrated voids volume fraction, temperature, displacement and stresses. Analytical results are verified by employing numerically analyzed results for unknown field functions and presented graphically for the vibrations of stress free field functions such as damping, frequencies and frequency-shift. The study of the chapter is based on three-phase-lag model of generalized thermoelasticity may receive better approach to allow voids and relaxation time parameters, which have many applications in the field of science, technology and engineering. The study may be useful in the area of seismology for mining and drilling in the earth's crust.

## Chapter-05

In this chapter the transversely isotropic electro-magneto nonlocal thermoelastic hollow sphere with voids material are addressed for free vibration analysis. By using time harmonics, stress-strain relations and modeling equations are transformed into ordinary differential equations and unknown field functions have been eliminated by using matrix elimination technique. In order to investigate the vibration analysis, the relations of frequency equations have been solved for assumed boundary conditions. To authenticate the phase-lag effects on the model of generalized thermoelasticity, the analytical results have been shown graphically in absence/presence of magnetic field. The magneto-thermoelastic solid materials with voids in respect of analysis of free vibrations have many applications such as designers of new materials in practical situations, acoustics, and oil prospecting etc.

## Chapter-06

In this chapter the vibrations of rigidly fixed electro-magneto nonlocal elastic voids cylinder with generalized thermoelasticity are discussed when the surfaces of nonlocal elastic hollow cylinder are assumed isothermal/thermally insulated and rigidly fixed. For the investigation of the vibrations of rigidly fixed boundaries, we make use of numerical Iteration method using MATLAB tool. Numerical computations in local/nonlocal elastic materials for free vibration field functions have been displayed graphically.

## Chapter-07

In this Chapter, conclusions of the present work, applications and its future scope are presented. The applications of the results obtained in previous chapters are presented in reference to different fields like: science, technology and engineering seismology for mining and drilling in the earth’s crust, designers of new materials in practical situations, acoustics, and oil prospecting etc.