CHAPTER-06

FREE VIBRATIONS OF RIGIDLY FIXED ELECTRO-MAGNETO NON-LOCAL GENERALIZED THERMOELASTIC CYLINDER WITH VOIDS MATERIAL

6.1 Introduction

This chapter concentrates on the vibrations of rigidly fixed electro-magneto nonlocal elastic voids cylinder with generalized thermoelasticity. The surfaces of nonlocal elastic hollow cylinder have been assumed isothermal/thermally insulated and rigidly fixed. For the investigation of the vibrations of rigidly fixed boundaries, we make use of numerical Iteration method using MATLAB tool. The real parts of generated data have been considered natural frequencies as shown in table. Numerical computations in local/nonlocal elastic materials for free vibration field functions have been displayed graphically.

6.2 The Basic Fundamental Equations and Mathematical Model

Here a transversely isotropic generalized thermoelastic cylinder/disk based on rigidly fixed boundary conditions with voids material has been presented with domain $a \le r \le a\eta$ having inner and outer radii $R_I = a,\ R_O = a\eta$ with nonlocal elasticity. Nonlocal elastic cylinder has been considered in undisturbed state at uniform temperature T_0 and perfectly conductive along z-axis. Therefore, the strength of magnetic field $\boldsymbol{H}$ proceeds in the direction of z-axis. The field components have been assumed as displacement vector $\boldsymbol{u} = (u_r, u_\theta, u_z)$ where $u_\theta = 0$, $u_z = 0, u_r = u(r,t)$, temperature component $T = T(r,t)$ and voids volume fraction $\varphi = \varphi(r,t)$. The strain components of hollow cylinder are $e_{rr} = \dfrac{\partial u}{\partial r}, e_{\theta\theta} = \dfrac{u}{r}, e_{zz} = 0,\ \ e_{r\theta} = e_{rz} = e_{\theta z} = 0$. The Maxwell's equations and the generalized Ohm's law have been generated by electro-magnetic field in the absence of charge density and displacement current as:

$$\begin{cases}\nabla\times \boldsymbol{E} = -(\partial \boldsymbol{B}/\partial t),\ \nabla\times \boldsymbol{H} = \boldsymbol{J},\\ \nabla.\boldsymbol{B}=0,\ \boldsymbol{B}=\mu_e \boldsymbol{H}\quad , \qquad\qquad ,\\ \boldsymbol{J}=\sigma(\boldsymbol{E}+\dot{u}\times \boldsymbol{B})\end{cases} \tag{6.1}$$

Here $\boldsymbol{J}$ is current density which is neglected because of minute effect of temperature gradient, the strength of magnetic field is $\boldsymbol{H}=\boldsymbol{h}+\boldsymbol{H}_0$, where h is perturbation of magnetic field which is so small that the product with displacement vector and their derivatives have been neglected due to linearization of basic equations and $\boldsymbol{H}_0=(0,0,H_0)$. Therefore the nonlocal stress-strain temperature relations and governing fundamental equations without heat sources, body forces and voids concentration have been presented (Cowin and Nanziato (1983) and Dhaliwal and Singh (1980)) as:

$$\frac{\partial \sigma_{rr}}{\partial r}+\frac{(\sigma_{rr}-\sigma_{\theta\theta})}{r}+\boldsymbol{F}_r=\rho(1-\zeta^2\nabla^2)\frac{\partial^2 u}{\partial t^2}, \tag{6.2}$$

$$\alpha\nabla^2\varphi-\left(\xi_1+\xi_2\frac{\partial}{\partial t}\right)\varphi-be+MT=\rho\chi(1-\zeta^2\nabla^2)\frac{\partial^2\varphi}{\partial t^2}, \tag{6.3}$$

$$\left(\frac{\partial}{\partial t}+t_0\frac{\partial^2}{\partial t^2}\right)\left(T_0\left(\beta_r\frac{\partial u}{\partial r}+\beta_\theta\frac{u}{r}\right)+MT_0\varphi\right)=\frac{1}{r}\frac{\partial}{\partial r}\left(rK\frac{\partial T}{\partial r}\right)-\left(\frac{\partial}{\partial t}+\delta_{1k}t_0\frac{\partial^2}{\partial t^2}\right)\rho C_e T, \tag{6.4}$$

$$\left.\begin{aligned}(1-\zeta^2\nabla^2)\,\sigma_{rr}&=\sigma_{rr}^L=c_{11}\frac{\partial u}{\partial r}+c_{12}\frac{u}{r}+b\varphi-\beta_r(T+\delta_{2k}t_1\dot{T})\\ (1-\zeta^2\nabla^2)\sigma_{\theta\theta}&=\sigma_{\theta\theta}^L=c_{11}\frac{u}{r}+c_{12}\frac{\partial u}{\partial r}+b\varphi-\beta_\theta(T+\delta_{2k}t_1\dot{T})\end{aligned}\right\}, \tag{6.5}$$

where $\zeta=e_0a_0$ is non local parameter, where a_0 is internal characteristic length and e_0 is material constant, $\sigma_{rr},\sigma_{\theta\theta}$ are radial, circular stress components. Here the quantities having superscript "L" stands for the local medium, also $\sigma_{ij}^L=\sigma_{ij};(i,j=r,\theta)$. $\boldsymbol{F}_r$ is the component of Lorentz force $\boldsymbol{F}=(\boldsymbol{J}\times\boldsymbol{B})$, ρ is mass density, β_r,β_θ are the thermal modulii, where $\beta_r=\beta_\theta=(c_{11}+c_{12})\alpha_1+c_{13}\alpha_3$; $\alpha_1=\alpha_3=\alpha_T$ is coefficient of linear thermal expansion (Dhaliwal and Singh (1980)),

C_e is specific heat at constant strain, K is thermal conductivity, T is the absolute temperature, T_0 is the reference temperature of the medium, t_0, t_1 are the thermal relaxation time parameter, φ is the void volume fraction field, α, b are the void parameters, χ is the equilibrated inertia, ξ_1, ξ_2 are the material constants due to presence of voids, M is thermo-void coupling parameter, $\boldsymbol{e}$ is dilatation, $\delta_{jk}; j=(1,2)$ is kronecker delta, in which $k=1$ represent LS theory and $k=2$ indicates GL theory of generalized thermoelasticity, c_{11}, c_{12} are elastic constants whose values are as given below:

$$c_{11} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \quad , \quad c_{12} = \frac{E\nu}{(1+\nu)(1-2\nu)} = c_{13},$$

where E and ν represent Young's modulus and Poisson ratio; $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}$ is Laplacian operator.

The thermally insulated/isothermal rigidly fixed boundary conditions have been considered with domain $a \le r \le a\eta$. Mathematically, we have:

$$\left.\begin{array}{l} T,_r = 0 \ , \ u = 0 \ , \ \varphi = 0 \\ T = 0 \ \ , \ u = 0 \ , \ \varphi = 0 \end{array}\right\} ; \ at \ r = a \ , \ a\eta \ , \tag{6.6}$$

6.3 Solution of Mathematical Model

The vibrations of thermoelastic cylinder is presumed to be restricted in radial direction, then using equations (6.1) in Lorentz force i.e. $F_r = (\boldsymbol{J} \times \boldsymbol{B})$ in radial direction (Das et al. (2013)), we obtained

$$F_r = \mu_e H_0^2 \left(\frac{u}{r} + \frac{\partial u}{\partial r} \right), \quad F_\theta = 0 \quad , \quad F_z = 0, \tag{6.7}$$

Substituting value of Lorentz force from equation (6.7) via equation (6.5) in equation (6.2), we get

$$\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}\right)+\frac{b}{c_{11}}\frac{\partial \varphi}{\partial r}-\frac{\beta_r}{c_{11}}\left(1+\delta_{2k}t_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial r}+\frac{\mu_e H_0^2}{c_{11}}\frac{\partial}{\partial r}\left(\frac{\partial u}{\partial r}+\frac{u}{r}\right) = \frac{\rho}{c_{11}}(1-\zeta^2\nabla^2)\frac{\partial^2 u}{\partial t^2} \quad . \quad (6.8)$$

Applying divergence both sides to equation (6.8) we get

$$R_h\nabla^2 e-\frac{\rho}{c_{11}}(1-\zeta^2\nabla^2)\frac{\partial^2 e}{\partial t^2}+\frac{b}{c_{11}}\nabla^2\varphi-\frac{\beta_r}{c_{11}}\left(1+\delta_{2k}t_1\frac{\partial}{\partial t}\right)\nabla^2 T=0, \quad (6.9)$$

where $e=\frac{u}{r}+\frac{\partial u}{\partial r}$, $R_h=1+\frac{\mu_e H_0^2}{c_{11}}$.

We now set up non-dimensional parameters given below

$$\left.\begin{array}{l}(u', r', \zeta_0)=\frac{1}{a}(u, r, \zeta), (\tau,\tau_0,\tau_1)=\frac{c}{a}(t, t_0, t_1), (\tau_{RR}, \tau_{\theta\theta})=\frac{1}{c_{11}}(\sigma_{rr}, \sigma_{\theta\theta}), \theta=\frac{T}{T_0}, \\ c_0=\frac{c_{12}}{c_{11}}, c=\sqrt{\frac{c_{11}}{\rho}}, \phi=\frac{\chi\Omega^{*2}}{a^2}\varphi, \bar{\xi}=\frac{c}{a}\frac{\xi_2}{\xi_1}, \bar{\beta}_R=\frac{\beta_r T_0}{c_{11}}, \bar{\beta}_\theta=\frac{\beta_\theta T_0}{c_{11}}, \bar{b}^*=\frac{a^2\bar{b}}{\chi\Omega^{*2}}, \bar{b}=\frac{b}{c_{11}},\end{array}\right\} . \quad (6.10)$$

Dashes have been suppressed for convenience. Using constant parameters from equation (6.10) in equations (6.3–6.5) and (6.9), following equations have been obtained

$$R_h\nabla_R^2 e-(1-\zeta_0^2\nabla_R^2)\frac{\partial^2 e}{\partial \tau^2}+\bar{b}^*\nabla_R^2\phi-\bar{\beta}_R\left(1+\delta_{2k}t_1\frac{\partial}{\partial \tau}\right)\nabla_R^2\theta=0, \quad (6.11)$$

$$\nabla_R^2\phi-a_1\left(1+\bar{\xi}\frac{\partial}{\partial \tau}\right)\phi-\frac{1}{\delta_1^2}(1-\zeta_0^2\nabla_R^2)\frac{\partial^2\phi}{\partial\tau^2}-a_2 e+a_3\theta=0, \quad (6.12)$$

$$\nabla_R^2\theta-\Omega^*\left(\frac{\partial}{\partial\tau}+\tau_0\frac{\partial^2}{\partial\tau^2}\right)\theta-\left(\frac{\partial}{\partial\tau}+\tau_0\frac{\partial^2}{\partial\tau^2}\right)(a_4 e+a_5\phi)=0, \quad (6.13)$$

where $a_1=\frac{\xi_1 a^2}{\alpha}, a_2=\frac{b\chi\Omega^{*2}}{\alpha}, a_3=\frac{M\chi\Omega^{*2}T_0}{\alpha}, a_4=\frac{\varepsilon_T\Omega^*}{\bar{\beta}_R}, a_5=\frac{cMa^3}{K\chi\Omega^{*2}}$,

$\omega^*=\frac{c_{11}C_e}{K}, \Omega^*=\frac{a\omega^*}{c}, \ \varepsilon_T=\frac{T_0\beta_r^2}{\rho C_e c_{11}}, \delta_1^2=\frac{\alpha}{\chi c_{11}}, \ e=\frac{1}{r}\frac{\partial}{\partial r}(ru), \nabla_R^2=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)$

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We introduce time harmonics as given below

$$(\bar{e} \quad \bar{\phi} \quad \bar{\theta}) = (e \quad \phi \quad \theta) exp(i\Omega\tau), \tag{6.14}$$

where $\Omega = \dfrac{\omega a}{c}$ is the circular frequency.

Using proposed time harmonics from above equation (6.14) in equations (6.11–6.13) we get

$$\left.\begin{array}{l} (\nabla_R^2 + A_{11})\ \bar{e} + A_{12}\nabla_R^2\bar{\phi} + A_{13}\nabla_R^2\bar{\theta} = 0 \\ -A_{21}\bar{e} \ + \ (\nabla_R^2 + A_{22})\ \bar{\phi} \ + \ A_{23}\bar{\theta} = 0 \\ A_{31}\bar{e} \ + \ A_{32}\bar{\phi} \ - \ (\nabla_R^2 - A_{33})\ \bar{\theta} = 0 \end{array}\right\}, \tag{6.15}$$

where $A_{11} = \dfrac{\Omega^2}{R_h - \zeta_0^2\,\Omega^2}, A_{12} = \dfrac{\bar{b}^*}{R_h - \zeta_0^2\,\Omega^2}, A_{13} = \dfrac{i\,\Omega\,\tau_1^*\bar{\beta}_R}{R_h - \zeta_0^2\,\Omega^2}$,

$A_{21} = \dfrac{a_2}{a_1^*},\ A_{22} = \dfrac{a_2^*}{a_1^*}, A_{23} = \dfrac{a_3}{a_1^*},\ a_1^* = 1 - \dfrac{\zeta_0^2\Omega^2}{\delta_1^2},\ a_2^* = a_1 i\Omega\bar{\xi}^* + \dfrac{\Omega^2}{\delta_1^2}, A_{31} = \Omega^2\tau_0^* a_4$,

$A_{32} = \tau_0^*\Omega^2 a_5\ ,\ A_{33} = \Omega^*\Omega^2\tau_0^*\ ,\ \tau_0^* = i\Omega^{-1} - \tau_0\ ,\ \tau_1^* = i\Omega^{-1} - \tau_1,\ \bar{\xi}^* = i\Omega^{-1} - \bar{\xi}$.

The unknowns $\bar{e},\ \bar{\phi}\ , \bar{\theta}$ have been eliminated from equation (6.15), for this non-trivial solution is appeared if the determinant of coefficient matrix vanishes and on expansion, we obtain:

$$(\nabla_R^6 - L\nabla_R^4 + M\nabla_R^2 - N)(\bar{e}\,,\ \bar{\phi},\ \bar{\theta}) = 0, \tag{6.16}$$

where $L = (A_{11} + A_{22} - A_{33} + A_{12}A_{21} + A_{13}A_{31})$, $N = (A_{11}A_{23}A_{32} - A_{11}A_{22}A_{33})$,

$M = (A_{22}A_{33} - A_{23}A_{32} + A_{11}A_{33} - A_{11}A_{22} + A_{12}A_{21}A_{33} + A_{12}A_{23}A_{31} - A_{13}A_{21}A_{32} - A_{13}A_{31}A_{22})$.

From the solution of equation (6.16), it has been analyzed that the solution is bounded for $r \to \infty$, for this there is a condition of the roots with positive real parts i.e. $\mathrm{Re}(k_i) \geq 0,\ \forall\ i = 1, 2, 3)$. Therefore, roots $k_i;\ i = 1, 2, 3$ of solution (6.16) have been achieved as given below:

$$k_1 = \sqrt{\frac{1}{3}\left(2p_1 \sin p_2 + L\right)},\ k_2 = \sqrt{\frac{1}{3}\left(L - p_1(\sqrt{3}\cos p_2 + \sin p_2)\right)},\ k_3 = \sqrt{\frac{1}{3}\left(L + p_1(\sqrt{3}\cos p_2 - \sin p_2)\right)},$$

where, $p_1 = \sqrt{L^2 - 3M}$, $p_3 = -\dfrac{2L^3 - 9LM + 27N}{2L^3}$, $p_2 = \dfrac{1}{3}\sin^{-1}(p_3)$.

On solving equation (6.16) Bessel form is obtained which may be written as

$$\begin{pmatrix} \bar{\theta} \\ \bar{e} \\ \bar{\phi} \end{pmatrix} = \sum_{i=1}^{3} \begin{pmatrix} 1 \\ R_i \\ S_i \end{pmatrix} \left[P_i J_0(k_i r) + Q_i Y_0(k_i r) \right], \tag{6.17}$$

where, $R_i = \dfrac{k_i^4 A_{13} + k_i^2 (A_{13}A_{22} - A_{12}A_{23})}{k_i^4 + (A_{11} + A_{22} + A_{12}A_{21})k_i^2 + A_{11}A_{22}}$, $S_i = \dfrac{k_i^2 A_{21} - (A_{21}A_{33} + A_{23}A_{31})}{k_i^2 A_{31} + A_{22}A_{31} + A_{21}A_{32}}$,

where P_i, Q_i; $i = 1, 2, 3$ are constants which depend on Ω only. J_0 and Y_0 are Bessel functions of order zero of First and Second kinds. Resolving cubical dilation $(\bar{e})$ from second part of equation (6.17) for displacement $\bar{u}$, we obtain

$$\bar{u} = \sum_{i=1}^{3} \frac{1}{k_i} R_i \left(P_i J_1(k_i r) - Q_i Y_1(k_i r) \right). \tag{6.18}$$

6.4 Frequency Equations

Substituting the equations (6.17) to (6.18) for rigidly fixed thermal boundaries in equation (6.6) at $r = 1$ and $r = \eta$. On simplification, we obtain

$$(\gamma_{ij})_{6\times 6}(P_1, P_2, P_3, Q_1, Q_2, Q_3)^T = 0 \;\; ; i, j = 1 \; to \; 6. \tag{6.19}$$

Equation (6.19) presents six algebraic linear homogeneous equations with six unknowns. For this a non-trivial solution is obtained if the determinant of matrix $(\gamma_{ij})_{6\times 6}$ vanishes which leads

$$\left| \gamma_{ij} \right| = 0 \; ; \quad i, j = 1 \; to \; 6, \tag{6.20}$$

where, the constant elements γ_{ij} ; $i, j = 1 \; to \; 6$ have been defined in thermally insulated/isothermal rigidly fixed conditions which are presented in two separate sets:

Set I: The elements of γ_{ij} ; $i, j = 1 \; to \; 6$ are given below

$$\left.\begin{aligned}&\gamma_{1j}=(R_iJ_1(k_i))/k_i,\ \gamma_{3j}=S_iJ_0(k_i),\ \gamma_{5j}=k_iJ_1(k_i);\ i,j=1,2,3\\&\gamma_{1j}=-(R_iY_1(k_i))/k_i,\ \gamma_{3j}=S_iY_0(k_i),\gamma_{5j}=-k_iY_1(k_i);i=1,2,3,\ j=4,5,6\end{aligned}\right\}, \quad (6.21)$$

Set II: For this set the elements of $\gamma_{ij}\ ;\ i,j=1to\ \ 6$, are given below

$$\left.\begin{aligned}&\gamma_{1j}=(R_iJ_1(k_i))/k_i\ ,\gamma_{3j}=S_iJ_0\left(k_i\right),\gamma_{5j}=J_0\left(k_i\right);\ i,j=1,2,3\\&\gamma_{1j}=-(R_iY_1(k_i))/k_i,\ \gamma_{3j}=S_iY_0\left(k_i\right),\gamma_{5j}=Y_0\left(k_i\right);\ i=1,2,3;\ j=4,5,6\end{aligned}\right\}. \quad (6.22)$$

The elements of $\gamma_{2j}, \gamma_{4j}, \gamma_{6j};\ j=1\ to\ 6$ are attained by inserting η along with k_i , in the elements of $\gamma_{1j}, \gamma_{3j}, \gamma_{5j};\ j=1\ to\ \ 6$.

6.5 Numerical Results and Discussion

In this section, the numerically analyzed computations have been done for the authentication of analytical results in transversely isotropic magneto-thermoelastic voids cylinder. The numerical results have been performed for coupled thermoelasticity (CTE), generalized thermoelasticity (GTE) and elasticity (E) for nonlocal and local thermoelastic hollow cylinder with radial thickness $\eta=2.0$. Modelling has been prepared for transversely isotropic thermoelastic solid with voids material single crystal of zinc whose physical constant values are given in SI units (Chadwick and Seet (1970))

$$\begin{aligned}&T_0=296K,\tau_0=0.05\times10^{-11}s,\ c_{11}=1.628\times10^{11}\,Nm^{-2},\ \alpha=3.688\times10^{-5}N,\\&\chi=1.753\times10^{-15}m^2,\ \ c_{12}=1.562\times10^{11}\,Nm^{-2},\ \ \xi_1=\xi_2=1.475\times10^{10}\,Nm^{-2},\\&K=1.24\times10^{2}Wm^{-1}\deg^{-1},\ C_e=3.9\times10^{2}\,JKg^{-1}\deg^{-1},\ \rho=7.14\times10^{3}Kg\,m^{-3},\ \omega=10,\\&M=2.0\times10^{6}\,Nm^{-2}\deg^{-2},\beta_r=\beta_\theta=5.75\times10^{6}\,Nm^{-2}\ \deg^{-1}\ ,\ b=1.13849\times10^{10}\,Nm^{-2}\end{aligned}$$

The magnetic field parameters have been assumed as $\mu_e=4\pi\times10^{7}H/m$, $H_0=10^{8}A/m$ from Othman and Hilal (2017). The nonlocal parameter has been assumed as $\xi_0=2.3102$. The secular dispersion relations have been obtained from assumed rigidly fixed boundaries are generally compound equations which give us the values in the form of complex numbers (real as well as imaginary part). The computations are applied to equation (6.20) for the cases of thermally insulated boundaries. The numerically generated complex values (frequencies) of Ω might be written as $\Omega^m = \Omega_R^m + i\Omega_I^m$. The real and imaginary parts have been presumed as natural frequencies $\Omega_R^m = \Omega_R$ and damping factor $\Omega_I^m = \Omega_I$ respectively. The value of m is presumed as mode number, corresponds to the root of equation (6.20). The real part of the roots of frequency is denoted as natural frequencies (Ω_R) have been shown in table 6.1 with nonlocal/local elastic materials at $\eta = 2.0$. It is noticed that initially values are low and from top to bottom, the trends of values go on increasing.

The frequency shift (Ω_{shift}) and the thermo-elastic damping related to inverse quality factor (Q^{-1}) for transversely isotropic electro-magneto generalized thermoelastic hollow cylinder have been calculated by Sharma et al. (2022a) as:

$$\Omega_{shift} = \left| \frac{\Omega_R^{\Upsilon^*} - \Omega_R^{CTE}}{\Omega_R^{CTE}} \right| , \quad Q^{-1} = 2\left| \frac{\Omega_I}{\Omega_R} \right| .$$

Frequency shift (Ω_R) versus mode number (m) is displayed in Fig. 6.1(a–b) for GTE and CTE models of magneto thermoelastic material in nonlocal/local elastic material with voids.

Table. 6.1: Natural frequencies (Ω_R) versus mode number for GTE, CTE and Elasticity models of magneto thermoelastic cylinder with voids at ratio of outer to inner radii $\eta = 2.0$ (a) nonlocal (b) local.

	Nonlocal thermoelastic cylinder			**Local thermoelastic cylinder**		
m	GTE	CTE	E	GTE	CTE	E
1	1.5265	1.4932	1.4149	1.7089	1.6814	1.6436
2	3.4556	3.4322	3.4141	3.5388	3.5222	3.4648
3	4.6468	4.6131	4.6051	4.7809	4.7431	4.7034
4	5.5131	5.4215	5.2931	5.8893	5.7797	5.5281
5	6.6359	6.5826	6.5543	6.8039	6.7708	6.6042

6	7.4622	7.4281	7.4001	7.8103	7.7455	7.6954
7	10.7103	10.5662	10.4457	10.9381	10.7791	10.6169
8	12.5516	12.4614	12.4017	12.5893	12.5088	12.4684
9	14.6091	14.5352	14.5032	14.5584	14.4839	14.4673
10	15.5836	15.4421	15.3431	15.7809	15.6998	15.6085
11	17.5727	17.4932	17.4312	17.8391	17.7069	17.6493
12	21.4311	21.3023	21.2819	21.7283	21.6978	21.6595

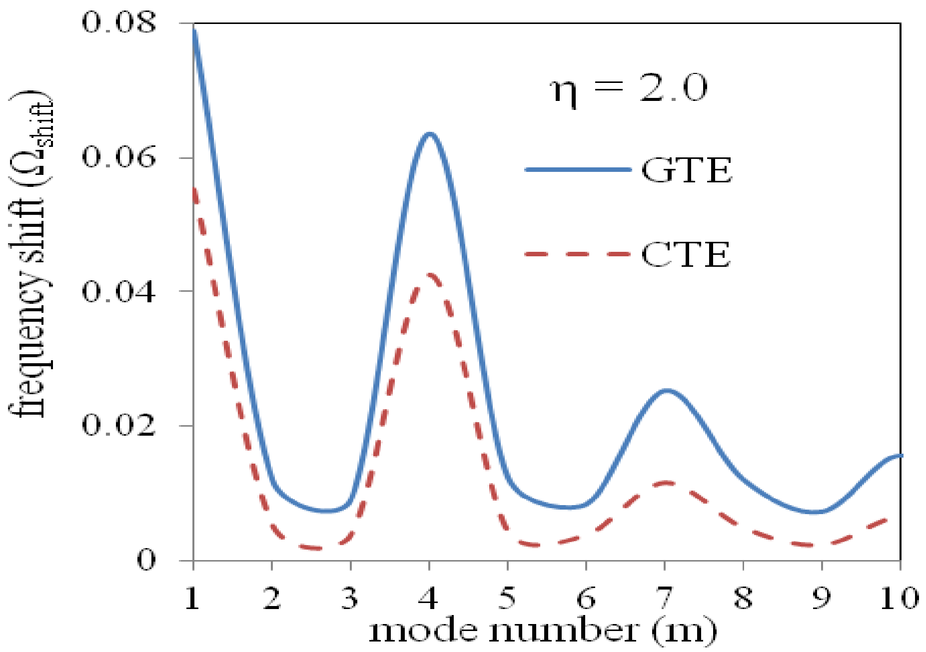

Figure 6.1(a): Frequency shift (Ω_{Shift}) versus mode number (m) at $\eta = 2.0$ for nonlocal case.

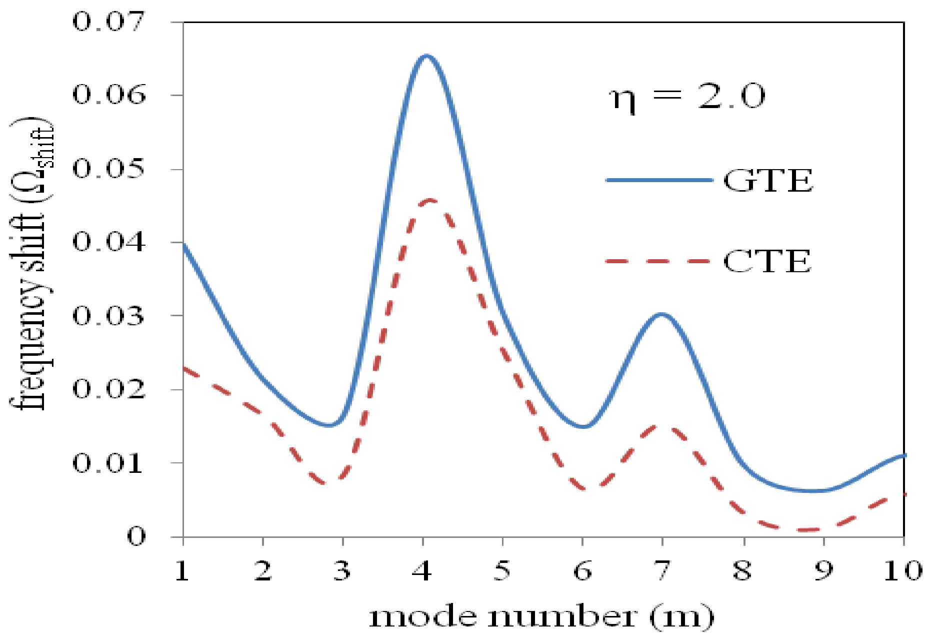

Figure 6.1(b): Frequency shift (Ω_{Shift}) versus mode number (m) at $\eta = 2.0$ for local case.

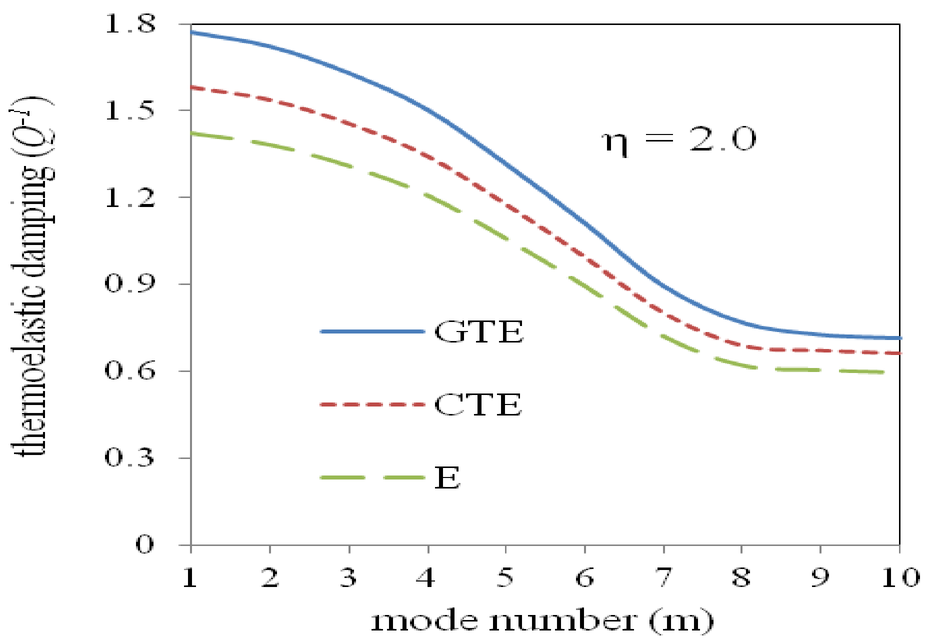

Figure 6.2(a): Thermoelastic damping (Q^{-1}) versus mode number (m) at $\eta = 2.0$ for nonlocal case.

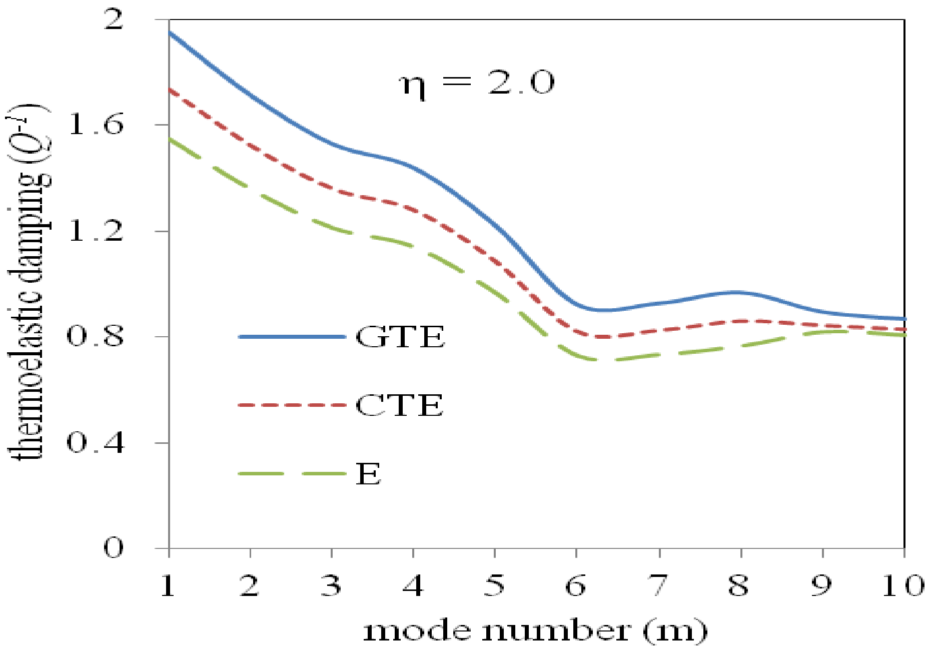

Figure 6.2(b): Thermoelastic damping (Q^{-1}) versus mode number (m) at $\eta = 2.0$ for local case.

It has been analyzed from Fig. 6.1(a-b) that initially the variations of frequency shift vibrations are larger and as to move from left to right the vibrations achieve peak values at $m = 4.0$, it go on decreasing. The peak values at $m = 4.0$ is larger in contrast to peak value at $m = 7.0$. The thermoelastic damping (Q^{-1}) versus mode number (m) is represented in Fig. 6.2(a-b) for GTE, CTE and elasticity (E) models of magneto thermoelastic cylinder in nonlocal/local elastic materials with voids. It is examined from Fig. 6.2(a-b) that initially the variations of frequency shift vibrations are larger and with increasing values of mode number, the variation of thermoelastic damping vibrations keep on decreasing to become linear at $m = 6.0$. This is to be noticed form graphical representation and table that the variation of vibrations is larger in case of GTE model in comparison to CTE and elasticity models.

6.6 Conclusions

Analysis of free vibrations of rigidly fixed boundary conditions has been investigated under LS and GL model of generalized electro-magneto-thermoelastic nonlocal cylinder with voids material. The vibrations of frequency relations are investigated analytically and numerically. The simulated results are presented graphically for frequency shift and quality factor. Also the real parts of frequencies named as natural frequencies have been shown in table 6.1. The graphical representation and table shows that the variation of vibrations is larger in case of GTE model in comparison to CTE and elasticity models. After achieving the maximum amplitude, the variations of frequency shift vibrations go on decreasing. From Fig. 6.2, it is revealed that the thermoelastic damping vibrations go on decreasing to become linear.